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Modeling binary stars: age, helium abundance, and convection parameters

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ABSTRACT

Stellar evolutionary models simulate well binary stars when individual stellar mass and system metallicity are known. The main goal of this paper is to determine a set of stellar parameters (mass, age, helium abundance, and convection parameters) for binary systems formed by FGK main-sequence stars of Population I. For a selected group of seven binaries, where luminosity, effective temperature, gravity, and metallicity are experimentally observed, we estimate the above mentioned set of stellar parameters (and provide estimation errors) by fitting simulated quantities to the observed ones. For each system, we estimate all such parameters and respective estimation errors, including individual masses (despite being known for these starts with an accuracy of 3%). The observational mass is recovered by the models with an absolute error of $0.02M_\odot$ (in average). Half of the stars considered have a mixing length parameter not compatible to the solar value, possibly due to the mild observed rotation of these stars. However, the error bars show that all these systems can be modeled using a solar helium-to-metal chemical enrichment parameter. Except for NGC188 KR 12, our results fit into what was obtained or predicted in previous works. Finally, we find a tendency for a decrease of the mixing length parameter and an increase of the overshooting with the increase of the mass. The analysis of these seven binaries and the corresponding fourteen stars shows that stellar solar models are not able to reproduce the observations in most of the cases, mainly due to the sub-solar mixing length parameter. We suspect that the obtained non-solar values may be caused by the absence of rotation in models.

Key words: binaries: general – fundamental parameters – low-mass – solar-type: stars.

1 INTRODUCTION

Binary stars offer several advantages for testing evolutionary models when compared to simple ones. Daniel Popper and Roger Ulrich published a paper in 1986 titled *"Can binary stars test solar models?"* (Popper & Ulrich 1986). They studied the main component of the Hyades eclipsing binary HD 27130, a star with a mass comparable to the one of the Sun. If the age and metallicity are assumed to be known (and equal to the cluster values), modeling this star resembles the solar case. Their main conclusion was that the solar models can reproduce the HR diagram position of the HD 27130 star, when the helium abundance is lower than the solar value. Five years later Andersen (1991), collected 45 eclipsing binaries observations and concluded that unless the errors in observed mass and radius are below 3%, such evolutionary models are not conclusive.

We do not recall here all the theoretical and observational arguments to support the claim that binary stars are fundamental to test stellar models. These arguments can be found in well known reviews such as Popper (1980), Andersen (1991), Lastennet & Valls-Gabaud (2002), Ribas (2006) or in the very recent papers Torres et al. (2010) and Southworth (2011). We can, however, summarize the main issues. First of all, binaries allow the determination of the star mass. If the binary is an eclipsing one, the radius for each star can also be calculated, and this can bring additional constraints on stellar models. As an example, we can point out the discrepancies among observations (mainly radii and effective temperature) for low-mass eclipsing binary stars (e.g., Morales et al. 2008). Secondly, in the case

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of eclipsing and spectroscopic binaries, the distance can be independently determined. This is particularly important for distances where trigonometric parallaxes are not sufficiently accurate (in the sense of a being longer than 100 parsecs (Perryman et al. 1995)). Thirdly, multiple systems are considered to be composed by coeval stars, i.e., stars with the same age and chemical composition, where a single isochrone must fit both stars. This canonical approximation can be supported by some theoretical and observational indications. The accretion rate, during the stellar formation, is mainly dependent on the stellar mass (White & Ghez 2001). Therefore, a binary component and a single star of identical mass have identical mass accretion rates. On the other hand, companions as close as 10 UA have little effect on the accretion phenomena. So, the formation of a binary star seems to be similar to the one of single stars. Moreover, Desidera et al. (2004) and Desidera et al. (2006) observed a metallicity difference between components of 56 wide binaries, using detailed spectroscopic analysis. However, they found that the differences are typically lower than 0.10 dex, and therefore it seems reasonable the traditional assumption of the same primordial chemical composition for both stars of a binary. This result is also supported by the analysis of the well known wide binary 16 Cygni, where the metallicity difference between both components is 0.02 dex (Ramírez et al. 2011).

If the mass, the metal abundance, and the position in the HR diagram for both components are known, then it is possible to determine fundamental parameters of the binary such as age and helium content (only available by direct observations for high mass stars) by means of theoretical stellar models. We must point out that this exercise, named *calibration of fundamental parameters*, is not possible for single stars except for the *Sun*. In general, for the majority of the cases there are more parameters to determine than available observations (e.g., Fernandes & Santos 2004).

It is known that a stellar theoretical model is dependent on many different variables: the stellar mass (M), the age ($t_\star$), the initial individual abundance of helium (Y) and metals (Z). When these variables are known, the internal stellar structure is fixed and the solution of the equations for the internal structure provide the values for the surface temperature, the bolometric luminosity, and the radius. Moreover, the integration of the stellar structure equations requires physical inputs to describe macro and micro physical processes inside the star. In particular, some poorly understood mechanisms such as convection, rotation, and diffusion are dependent on free parameters. For instance, in the framework of Mixing Length Theory (MLT, see below) currently used to model stellar convection, two (unknown) variables must be considered: the mixing-length parameter (α) and the amount of overshooting in the convective core (ov).

From the observational point of view, a detailed spectroscopic analysis of a star allow us to estimate its metallicity ($[Fe/H]$ or Z/X, where $X = 1 - Y - Z$ is the abundance of hydrogen), the effective temperature ($T_{\rm eff}$), and the gravity (considered as log g). Currently, the knowledge of the photometry (including bolometric correction) and parallax allows the luminosity (L) to be determined. We thus have four known observable parameters.

On the other hand, we face six unknowns, i.e., M, $t_\star$, Y, Z, α, and ov. A possible consequence of having six unknowns with respect to four observable parameters is a certain underdeterminacy of the stellar parameters, meaning that one can have an infinite number of combinations of M, $t_\star$, Y, Z, α, and ov fitting the four observations. This underdeterminacy problem is currently solved by assuming, in the models, the solar values for the helium-to-metal chemical enrichment parameter $\frac{\Delta Y}{\Delta Z}$ and for the mixing length parameter. However, these approximations are not necessarily supported by observations. The value in $\Delta Y/\Delta Z = (\frac{Y-0.23}{Z})$ can vary among nearby FGK stars (e.g., Jimenez et al. 2003; Casagrande et al. 2007; Bertelli et al. 2008) and among clusters (Portinari et al. 2010). Furthermore, different studies have shown that solar-scaled values for the helium and the mixing length parameter are not adapted to the nearby visual binary star α *Centauri* and for the Hyades (Gennaro et al. 2010; Casagrande et al. 2007; Eggenberger et al. 2004; Miglio & Montalbán 2005; Lebreton et al. 2001; Yildiz 2007). As for the possible uniqueness of the overshooting parameter, for a value around $0.20H_p$ or $0.25H_p$, one can see contradictory results for double-line eclipsing binary systems, either with a constant overshooting (Claret 2007) or with a variable one dependant on the stellar mass (Ribas et al. 2000).

Recently, we have presented a mathematical optimization methodology to estimate the stellar parameters (mass, age, helium, and mixing length parameter) from the corresponding $T_{\rm eff}$, L, $[Fe/H]$, and g, and applied it to 115 FGK main-sequence stars of Population I issued from detailed spectroscopic analysis (Fernandes et al. 2011). In the current work, we apply this methodology to individual stellar members of binary systems, determining also the metal abundance and the overshooting. For this purpose, we select seven binary systems with known metallicity from the recent compilation available in Torres et al. (2010), where components are FGK main-sequence stars of Population I. The individual masses of these binary components exhibited an accuracy of at most 3%.

Our paper is organized as follows. The observational data is discussed in Section 2, where we describe the input physics chosen for our work and recall the optimization methodology. We then report and discuss in Section 3 the results for the selected binary systems. Finally, in Section 4, we draw some conclusions and suggest future work.

2 OBSERVATIONAL DATA AND METHODOLOGY

The binaries where selected from Torres et al. (2010) taking into account the following criteria. Components A and B are FGK main-sequence stars with known high precision mass (varying between $0.8M_\odot$ and $1.3M_\odot$) for which the metallicity values are available. Combining Tables 2 and 3 in Torres et al. (2010), we selected seven systems among the 95 binaries considered in that paper. The correspondent observational data for the effective temperature, gravity, luminosity, and Z/X is presented in Table 1, where the corresponding errors for the observations are also included. The value of Z/X for each star was computed from the observed $[Fe/H]$ and $(Z/X)_\odot = 0.0245$ (Grevesse & Noels 1993). We would like to point out that in our sample we have systems with subsolar metallicity values. It is known that for low metallicity stars the abundance of heavy elements cannot be simply derived from $[Fe/H]$, due to the overabundance of the so called α elements. Salaris et al. (1993) derived a correction formula to be applied to subsolar metal stars, however, the resulting effect for Population I stars is marginal. In particular, our poorest binary is WZ Oph (HD 154676), with $[Fe/H] = -0.27 \pm 0.07$, and the correspondent Z correction output is, approximately, 4%. Moreover, the observational error in Z is 16% and therefore we decided not to apply the correction.

Our list is composed basically by nearby eclipsing binaries ($d < 200$ *pc*), with two exceptions: the visual binary *α Centauri* and the eclipsing binary GSC 04619-00585, the latter a member of the open cluster NGC188 at a distance close to 2000 *pc*. Binary *α Centauri* is the nearest stellar system located at a distance of 1.3 *pc*, and it is the only visual binary included in the list of Torres et al. (2010) due to very accurate masses: in the abstract they curiously point out that: *"We have identified 95 detached binary systems containing 190 stars (94 eclipsing systems, and α Centauri) that satisfy our criterion that the mass and radius of both stars be known to ±3% or better"*. Both stars, *α Cen A* and *α Cen B*, are in the main sequence and have a typical solar mass value. Being a visual and spectroscopic binary, with a period inferior to 80 years and a separation between components of about 40 UA, the orbital parameters and individual masses are very well known. The individual $T_{\rm eff}$ and metallicity for both stars have been intensively measured by spectroscopy in the last decade. Ground based observations have allowed individual radii (using the VLTI) and asteroseismic properties for both components to be known. Thus, masses, radii, and $T_{\rm eff}$ for both stars are known with an accuracy as good as 1%. After the *Sun*, these two stars are certainly the best known of all. Therefore, α Centauri is a real benchmark for comparisons between observations and stellar models. A complete overview on the binary modeling is given in the papers (Eggenberger et al. 2004; Miglio & Montalbán 2005; Yildiz 2007). On the other hand, NGC188 KR V12 in composed by two very similar stars with solar masses ($1.103 M_\odot$ and $1.081 M_\odot$) near the end of their main sequence. This fact is particularly interesting in order to determine the cluster age (Meibom et al. 2009). Additionally, some of the remaining systems are also interesting as some authors reported difficulties to model them with solar scaled models: Clausen et al. (2009) concludes that both components of the eclipsing binary V636 Cen should have mixing length parameters significantly lower than the solar value in order to reproduce the observations; this seems also to be the case for the components of the eclipsing binary WZ Oph (Clausen et al. 2008); according to Lastennet & Valls-Gabaud (2002), for the solar metal binary UX Men, the observed HR diagram position of both components could be reconciled with models if a super solar helium abundance is considered valid.

2.1 CESAM and PSwarm: the stellar modeling optimization methodology

The optimization methodology is presented in detail in Fernandes et al. (2011) and we only recall here some of the key issues. Our methodology is based on the interface between the PSwarm algorithm, which has been recently shown to perform well in a large variety of bound-constrained optimization problems (Vaz & Vicente 2007), and the stellar evolutionary code CESAM, version 3 (Morel 1997).

We considered the following parameters for estimation: the stellar mass M ($M_\odot$), the abundances of hydrogen X, helium Y, and metals Z, the stellar age $t_\star$ (Myr), the stellar surface convection α, and the stellar nucleus overshooting[1] ov (H_p). Because X, Y, and Z are related by $Z = 1 - X - Y$, only two of these variables need to be considered, and we chose to work with X and Y.

For a given star, the evolution simulation is performed for specific values of these six parameters (M, $t_\star$, X, Y, α, and ov). Among other features, we obtain from the simulation the effective surface temperature $T_{\rm eff}$, the luminosity L, and the radius R (from which we can then obtain the stellar gravity by $g = 27397 M/R^2$). From the observational point of view, we have an estimate for the metalicity Z/X, the luminosity, the effective temperature, and the gravity. Absolute errors are available for all these observed quantities.

We are therefore left with four observed quantities and six unknown stellar parameters. The proposed optimization methodology consists in the computation of the six unknown stellar parameters by solving a minimization problem, whose objective function reflects the fit between the simulated and observed stellar features. The minimization problem is thus defined by

$$\min_{M,t_\star,X,Y,\alpha,ov} \left(\frac{T_{\rm eff}-T_{{\rm eff},obs}}{\delta T_{{\rm eff},obs}}\right)^2 + \left(\frac{L-L_{obs}}{\delta L_{obs}}\right)^2 + \left(\frac{\frac{Z}{X}-\left(\frac{Z}{X}\right)_{obs}}{\delta\left(\frac{Z}{X}\right)_{obs}}\right)^2 + \left(\frac{g-g_{obs}}{\delta g_{obs}}\right)^2, \quad (1)$$

subject to

$$(\underline{M}, \underline{t_\star}, \underline{X}, \underline{Y}, \underline{\alpha}, \underline{ov}) \leqslant (M, t_\star, X, Y, \alpha, ov) \leqslant (\overline{M}, \overline{t_\star}, \overline{X}, \overline{Y}, \overline{\alpha}, \overline{ov}), \quad (2)$$

where $Z = 1 - X - Y$ and the subscript *obs* and the prefix δ denote the observed data and the corresponding absolute errors, respectively. The vectors $(\underline{M}, \underline{t_\star}, \underline{X}, \underline{Y}, \underline{\alpha}, \underline{ov})$ and $(\overline{M}, \overline{t_\star}, \overline{X}, \overline{Y}, \overline{\alpha}, \overline{ov})$ represent lower and upper bounds on the variables, respectively.

The lower and upper bounds considered in (2) are reported in Table 2. These bounds were chosen to be representative of nearby FGK main-sequence stars of Population I.

Note that the objective function (1) is nonlinear in the optimization variables (M, $t_\star$, X, Y, α, and ov), because its evaluation depends on the solution of the underlying stellar evolution equations. We face an inverse or parameter estimation problem of the simulation-based type, for which the objective function is expensive to evaluate and its derivatives are unavailable. Our numerical experience has also shown that this problem has non-unique global minimizers. Thus, to properly solve it, one must select a solver capable of global optimization without the use of derivatives (and, preferably, in a parallel environment, given the cost of the numerical simulations).

As input physics for the stellar evolutionary code we

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[1] ov (H_p) is assumed zero if $M < 1.1 M_\odot$.

Table 1. Observational data for the selected seven binary systems according to Torres et al. (2010).

Star	$T_{\rm eff,\it obs}(K)$	$\delta T_{\rm eff,\it obs}(K)$	$g_{obs}(m/s^2)$	$\delta g_{obs}(m/s^2)$	$L_{obs}(L_\odot)$	$\delta L_{obs}(L_\odot)$	Z/X_{obs}	$\delta Z/X_{obs}$
VZ Hya A (HD 72257)	6645	150	20184	139	3.02	0.27	0.0155	0.0043
VZ Hya B	6290	150	25351	350	1.74	0.17	0.0155	0.0043
V505 Per A (HD 14384)	6510	50	21038	775	2.67	0.13	0.0186	0.0013
V505 Per B	6460	50	21429	839	2.51	0.12	0.0186	0.0013
UX Men A (HD 37513)	6200	100	18621	386	2.41	0.16	0.0269	0.0062
UX Men B	6150	100	20184	418	2.09	0.14	0.0269	0.0062
WZ Oph A (HD 154676)	6165	100	17140	276	2.55	0.17	0.0132	0.0021
WZ Oph B	6115	100	16596	267	2.53	0.17	0.0132	0.0021
α Cen A (HD 128620)	5824	26	20230	186	1.55	0.03	0.0426	0.0039
α Cen B (HD 128621)	5223	62	34356	475	0.50	0.02	0.0426	0.0039
NGC188 KR V12 A	5900	100	14894	412	2.21	0.16	0.0195	0.0040
NGC188 KR V12 B	5875	100	15704	434	2.02	0.15	0.0195	0.0040
V636 Cen A (HD 124784)	5900	85	27797	256	1.13	0.07	0.0155	0.0028
V636 Cen B	5000	100	34041	314	0.39	0.03	0.0155	0.0028

Table 2. Lower and upper bounds on the variables.

	M ($M_\odot$)	$t_\star$ (Myr)	X	Y	α (H_p)	ov (H_p)
Lower	0.7	100	0.500	0.23	0.5	0.0
Upper	1.4	9999	0.761	0.35	2.5	0.5

chose the following. The nuclear reactions rates were taken from Caughlan & Fowler (1988). The OPAL opacities were the ones from Iglesias & Rogers (1996) combined at low temperatures with data from Alexander & Ferguson (1994), following the approach in Houdek & Rogl (1996). The atmosphere was described by an Eddington $T(\tau)$-law. The convection was treated according to the MLT from Böhm-Vitense (1958), leaving the mixing length parameter (proportional to the pressure scale height H_p) as an unknown, and thus letting α and ov be free parameters. The equation of state was chosen as the so-called EFF (Eggleton et al. 1973), which is valid for solar-type stars where the departure from the ideal gas is not relevant (note that due to the large amount of computation required in our work, an analytical equation of state is clearly more suitable than a tabulated one). Finally, the solar mixture was taken from Grevesse & Noels (1993). Using this input physics, we were able in Fernandes et al. (2011) to reproduce to within five digits of accuracy the observed solar luminosity, radius, and metallicity for an helium abundance of 0.28±0.01, metal abundance of 0.0173±0.0010, and mixing length parameter of 1.6 (±0.1)H_p. With these values we obtained $(\frac{\Delta Y}{\Delta Z})_\odot \sim 2.8\pm1.0$. **We would like to point out that the most recent studies about helium primordial abundance seem to indicate a value around 0.25 (Izotov & Thuan 2010) higher than the one used in this work (0.23). However, in order to keep the consistency with our previous work presented in Fernandes et al. (2011), we chose to keep the lower value. Anyhow we should stress that a higher helium value has an impact on the solar helium-to-metal chemical enrichment parameter which then becomes 1.7.**

We applied our simulation-based optimization procedure (CESAM-PSwarm) to each star of each system to estimate M, $t_\star$, X, Y, α, and ov, treating the stars of the binary as individual objects. Our choice to include the mass as a free parameter aims at comparing the estimated values to the observed ones, in order to check for the accuracy of our method. As already mentioned, there is no unique solution to the corresponding optimization problem since the number of parameters to be estimated is larger than the number of available observations. Additional difficulties arise from the high non-linearity of the stellar evolution equations and the use of relatively small observational error boxes compared to the feasible region formed by the bounds on the optimization variables.

To overcome these difficulties, we ran the simulation-based optimization 35 times for each individual star. Thus, for each binary, we have 35 × 35 pairs of runs. To assess the agreement between stars in the same binary we computed the following metric:

$$\theta \;=\; \left(\frac{t_{A\star}-t_{B\star}}{500\ Myrs}\right)^2+\left(\frac{X_A-X_B}{0.01}\right)^2+\left(\frac{Y_A-Y_B}{0.01}\right)^2, \quad (3)$$

where the subscripts A and B refer to primary and secondary binary members. Pairs of runs for which $\theta < 1$ indicate that a match is found with an accuracy of 500 Myrs in age, and 0.01 in helium and hydrogen abundances. We have thus excluded from the set of 35 × 35 pairs of runs those for which $\theta \geqslant 1$. In addition, it happened in some cases that the objective function value at the solution found was greater than or equal to 1, indicating that a match between the observed and estimated values may not have been achieved. We have further excluded all the pairs of runs for which the objective function value was greater than or equal to 1 for at least one of the binary members.

In Table 3 we report the obtained solutions for the six parameters. For each binary member, the reported values correspond to the average of the estimated values for the pairs of runs not excluded, i.e., for the pairs of runs for which the matching between binary members was acceptable ($\theta < 1$) and the objective function value was less than 1 for both binary members.

The absolute errors for the original estimated parameters, reported in Table 3, were obtained by performing additional estimations. In these additional estimations we considered the individual observational parameters perturbed by the extreme values of the corresponding observed absolute errors. For example, the observed mass parameter M_{obs} was perturbed by considering $M_{obs} \pm \delta M_{obs}$, while the remaining observed parameters were kept at their previous values. Therefore, we have performed 8 additional estimations (2 for each observed parameter) per star. In order to keep the number of optimization runs at a reasonable size we reduced to 10 the number of runs for each estimation instance (yielding a final number of $8 \times 14 \times 10 = 1120$ optimization runs). Given the limited number of optimization runs, imposing the pair matching described in (3) would result in a small number of runs for computing the average. We have thus used all the runs which gave an objective function value lower than 1 in the average computation. The errors reported in Table 3 correspond to the root mean square of the absolute errors between the estimated perturbed parameters and the original estimated ones. For instance, for the mass, we used the formula

$$\sqrt{\frac{\sum_{i=1}^{8}\left(M^{*}-\overline{M}_{i}^{*}\right)^{2}}{8}},$$

where M^* is the original estimated mass and $\overline{M}_i^*$ is the estimated mass in the i-th perturbed case (average of the corresponding 10 optimization runs with an objective function value lower than 1).

In Table 4 we report these estimation errors in a different way, for each observed parameters separately. For each observed parameter and for extreme value (up and down) of the corresponding observed absolute errors, we first averaged the perturbed estimated parameter using the 10 optimization runs (with an objective function value lower than 1). Then we averaged the obtained absolute error for the two extremes and the 14 starts. For instance, for the observed parameter g and the estimated parameter M, we used the formula

$$\frac{\sum_{s=1}^{14}\left|M_s^{*}-\overline{M^{+}}_s^{*}\right|+\sum_{s=1}^{14}\left|M_s^{*}-\overline{M^{-}}_s^{*}\right|}{28},$$

where, for each s-th star, M_s^* is the original estimated mass and $\overline{M^{+}}_s^{*}$ and $\overline{M^{-}}_s^{*}$ are the estimated masses, perturbing g respectively above and below (in all three cases averaging the corresponding 10 optimization runs with an objective function value lower than 1). The star WZ Oph B was removed from this calculation in the $T_{\rm eff}$ case, since no run with an objective function value lower than 1 was found.

3 DISCUSSION OF THE RESULTS

First we compare the masses estimated by our optimization methodology to those reported in Torres et al. (2010). For this purpose, we depict in Figure 1 a comparison between the results obtained by Torres et al. (2010) and the ones obtained by us, together with the respective error bars (our estimated values and errors are those of Table 3). The estimations match each other well, as it can be confirmed by the linear regression included in the picture. This is not a surprising result since the optimization method involves simultaneously the gravity, the luminosity, and the effective temperature, and, in addition, we have used in our results those optimization runs for which the final objective function value was lower than 1 (for both stars in the binary system) and that led to a pair matching ($\theta < 1$) in the binary system.

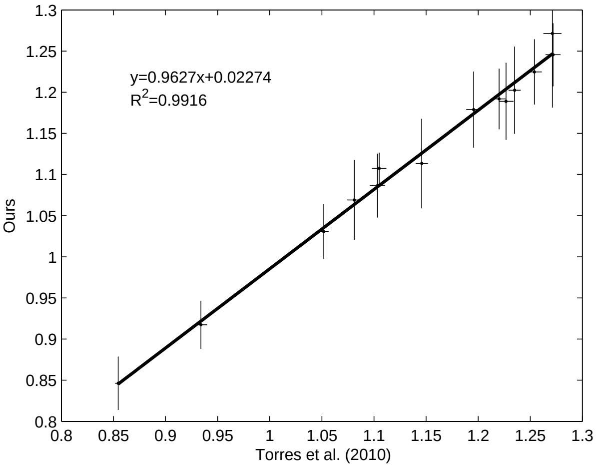

Figure 1. Torres et al. (2010) results *vs* our estimated mass results.

3.1 Discussion for all binaries

We now discuss the obtained results for all binary systems considered.

- **VZ Hya**. We obtained an age around 1 Gyr, in close agreement with Clausen et al. (2008) who found values from 0.75 to 1.25 Gyr, depending on the source of the evolutionary tracks. Considering the obtained estimation errors, the mixing length parameter of both components and $\frac{\Delta Y}{\Delta Z} \sim 3.5 \pm 3.5$ are solar values.
- **V505 Per**. We predicted that this system is very young, with an age of 0.7-0.8 Gyr, in a relatively good agreement with Tomasella et al. (2008), who found 0.9 Gyr. However, as in the VZ Hya, the mixing length parameters are subsolar and the $\frac{\Delta Y}{\Delta Z}$ is higher than the solar value ($\sim 4 \pm 1.5$).
- **UX Men**. We found an age of 2.1 Gy, in a good agreement with Lastennet & Valls-Gabaud (2002), who found it varying from 1.8 to 2.5 Gyr. They recognized the difficulties in reproducing the observed metallicity for this star: "*we suspect that a slight increase in the Helium abundance may reconcile the range inferred from its position on the HR diagram with the spectroscopically determined metallicity*". Such a statement is compatible with our results, as we found an helium abundance ($\frac{\Delta Y}{\Delta Z} \sim 3.5 \pm 2.0$) higher than the corresponding solar value.
- **WZ Oph**. This binary is also considered in Clausen et al. (2008) where no success is achieved in reproducing the observations from the stellar evolutionary tracks. These authors argued that "*models are significantly hotter than ob-*

Table 3. The obtained solutions for mass, age, hydrogen, helium, mixing length, and overshooting for each binary member. The corresponding metal abundance is also reported (Z). In parentheses we report the absolute errors in the estimation.

Star	M ($M_\odot$)	$t_\star$ (Myr)	X	Y	Z	α (H_p)	ov (H_p)
VZ Hya A (HD 72257)	1.27(±0.09)	1080(±673)	0.719(±0.036)	0.269(±0.036)	0.013	1.53(±0.35)	0.09(±0.11)
VZ Hya B	1.11(±0.05)	910(±486)	0.717(±0.029)	0.273(±0.029)	0.010	1.46(±0.15)	0.26(±0.17)
V505 Per A (HD 14384)	1.25(±0.04)	702(±464)	0.705(±0.019)	0.282(±0.019)	0.013	1.30(±0.18)	0.38(±0.10)
V505 Per B	1.22(±0.04)	831(±623)	0.705(±0.018)	0.282(±0.018)	0.013	1.35(±0.19)	0.33(±0.08)
UX Men A (HD 37513)	1.20(±0.05)	2052(±762)	0.686(±0.023)	0.296(±0.023)	0.019	1.29(±0.20)	0.24(±0.07)
UX Men B	1.18(±0.05)	2120(±1003)	0.686(±0.021)	0.295(±0.021)	0.019	1.42(±0.21)	0.36(±0.08)
WZ Oph A (HD 154676)	1.19(±0.05)	2676(±878)	0.744(±0.015)	0.246(±0.015)	0.010	1.00(±0.27)	0.29(±0.06)
WZ Oph B	1.19(±0.04)	2847(±748)	0.750(±0.012)	0.240(±0.012)	0.010	0.89(±0.22)	0.28(±0.06)
α Cen A (HD 128620)	1.11(±0.02)	5034(±962)	0.672(±0.011)	0.300(±0.011)	0.028	1.64(±0.13)	0.22(±0.09)
α Cen B (HD 128621)	0.92(±0.03)	5053(±961)	0.674(±0.018)	0.297(±0.018)	0.029	1.82(±0.14)	0.00(±0.01)
NGC188 KR V12 A	1.09(±0.04)	3662(±567)	0.687(±0.015)	0.299(±0.016)	0.014	0.88(±0.16)	0.11(±0.13)
NGC188 KR V12 B	1.07(±0.05)	3579(±716)	0.685(±0.022)	0.301(±0.022)	0.013	0.90(±0.15)	0.05(±0.13)
V636 Cen A (HD 124784)	1.03(±0.03)	3052(±905)	0.737(±0.018)	0.252(±0.018)	0.012	1.57(±0.11)	0.00(±0.21)
V636 Cen B	0.85(±0.03)	3534(±1132)	0.738(±0.021)	0.251(±0.021)	0.011	1.07(±0.04)	0.00(±0.00)

Table 4. Variation on the estimated parameters versus the variation on the observed parameters.

	M ($M_\odot$)	$t_\star$ (Myr)	X	Y	α (H_p)	ov (H_p)
g	0.02	576	0.011	0.012	0.13	0.09
L	0.06	706	0.020	0.020	0.16	0.07
$T_{\rm eff}$	0.05	696	0.024	0.025	0.17	0.07
Z/X	0.02	535	0.013	0.012	0.15	0.10
Average	0.04	628	0.017	0.017	0.15	0.08

served. We tentatively conclude that this is caused either by WZ Oph anomalies such as a low He content or decreased envelope convection, or by underestimated interstellar reddening". In fact, our results indicate a low helium abundance (0.24-0.25) and low values for both mixing length parameters.

- α **Centauri**. We found an age of 5 Gyr in close agreement with previous estimations. In the literature, one can find values between 5 and 6 Gyr, with an internal error of 0.5 Gyr, depending on the authors (Eggenberger et al. 2004; Miglio & Montalbán 2005; Yildiz 2007). We found a difference between the values of the mixing length parameter for both stars in agreement with previous results. However, our results indicate that the mixing length parameter of *α Cen B* is higher than the one of *α Cen A*, in agreement with Eggenberger et al. (2004) and Miglio & Montalbán (2005), but in disagreement with Yildiz (2007). Our results indicate an overshooting of 0.22 H_p for α Cent A. The eventual existence of a convective core in α Cent A has been a reason of controversy in the literature during the last years. Our results, as well as the ones of other authors (Eggenberger et al. 2004; Miglio & Montalbán 2005), predict a convective core with an overshooting value typical for what is observed in eclipsing binary members of equal mass Claret (2007). Nevertheless, the recent results of Meulenaer et al. (2010) obtained from an asteroseismic analysis (using large and small separations) seem to be in favor of a radiative core. Finally, we found $\frac{\Delta Y}{\Delta Z} \sim 2.5 \pm 1.0$, basically the solar value.

- **NGC188 KR V12**. We obtained an age near 3.6 Gy and a helium higher than the solar value, and the mixing length parameters for both stars were clearly subsolar. In order to analyze these results, we compared them to the recent work of Meibom et al. (2009), who performed a detailed observational and theoretical analysis of this binary. Using different diagram plans (Color-Magnitude, radius *vs* $T_{\rm eff}$ and mass *vs* luminosity) they concluded that models with solar metallicity can reproduce better the different observations than $[Fe/H] = -0.1$ (the value assumed by us in our work). Their Figure 10 is illustrative of their conclusion: subsolar metal isochrone is about 100 K to 200 K hotter than the binary Color-Magnitude diagram position. So, the authors suggest that "*plausible changes in the model metallicity and distance to better match the isochrones to the cluster sequences*". Our results propose an alternative solution: it is possible to recover the observations by means of the theoretical models, if the mixing length parameter for the binary models are reduced below 1 and the helium is increased ($\frac{\Delta Y}{\Delta Z} \sim 5.0 \pm 2.5$). Their derived age is 6.2 ± 0.2 higher than our quoted value.

- **V636 Cen**. We determined an age of about 3 Gyr and a solar value $\frac{\Delta Y}{\Delta Z} \sim 2.0 \pm 2.0$. The mixing length for V636 Cen A was near the solar value, in opposition to its companion. This binary was recent reviewed by Clausen et al. (2009). They found an age of 1.35 Gyr and different mixing lengths parameters for both stars: 1.4 (primary) and 1.0 (secondary). Both results can be considered in agreement with ours.

3.2 Mixing length parameter and overshooting: rotation and mass dependence

Among the 14 members, 9 of them show mixing length parameters clearly lower than the solar value. Thus, we must verify if our method creates an artificial bias to low values of the mixing length parameters. The methodology used in this paper was previously applied to 115 stars (Fernandes et al. 2011). These are also FGK main-sequence stars of Population I: 51 stars have mixing length parameters of 1.6 ± 0.2 (typically the solar value); 37 stars have values below 1.3, and 32 have values larger than 1.9. So, we do not have reason to believe in a low mixing length parameter bias.

Having 14 independent estimations of the mixing length parameter, we would like to take this opportunity to discuss in more detail our results. A global discussion of the mixing length parameter, in the framework of previous studies, is rather difficult because the majority of the modeling approaches do not allow the mixing length to be a free parameter. Frequently, a value of the mixing length parameter equal to the solar one or near it (within discrepancies lower than $\Delta\alpha = 0.2$) seems appropriated. However, as we pointed out in the Introduction, this is not always the case, as we can see for Hyades. But there are other cases. Standard models can reproduce observations for UV Psc only if the mixing length parameter of both components is lower than 1.0 (Lastennet et al. 2003). Torres et al. (2006) have analyzed the eclipsing binary V1061 Cygni (1.2 and 0.9 $M_{\odot}$) and they found the solar mixing length parameter for the main component, but $\alpha = 1.0$ for the V1061 Cygni B. Pre-main sequence binaries seem to be fitted better with models with mixing length parameter of 1.0 instead of the solar value (Hillenbrand & White 2004). These cases are sufficient to illustrate that non-solar mixing length parameters values can be considered (see Morales et al. (2009, 2010) for more details). It is very well known that the MLT is a crude representation of real convection.

We would like to recall that a change in the mixing length parameter has a direct effect on the convective efficiency in the super-adiabatique layer of the external convective region. The effect of the mixing length parameter is mainly in the external regions of the star, affecting the radius and the $T_{\rm eff}$. The luminosity (in the main sequence) is slightly changed. Such possible discrepancies of the mixing length parameter (relative to the solar values) may be due to the inaccuracies in the treatment of the atmosphere of the star, in the stellar activity, in the rotation, or even in all of them together (e.g., Chabrier et al. 2007; Torres et al. 2006). In Figure 2 we plot the mixing length parameter *versus* the rotational velocity ($v\ sin\ i$) for the binary components analyzed in this work. For the sake of illustration, we plot also the *Sun*, marked by a star. The figure seems to indicate that slow rotators have higher mixing length parameters. Our results are supported by others for visual binary members η Casssiopeiae A (Fernandes et al. 1998), 70 Ophiuchi A (Fernandes et al. 1998), ξ Bootis A (Fernandes et al. 1998), and 85 Pegasi (Fernandes et al. 2002) (in which the CESAM code as also been used). These stars are slow rotators ($v\ sin\ i < 6\ km/s$) and have solar mixing length parameters. This is also in agreement with what has been found for the eclipsing binary UV Piscium (referred above) which has exhibited mixing length parameters clearly sub-solar and $v\ sin\ i$ larger than 50 km/s for both stars. So, the rotation could be a plausible reason for the low mixing lengths parameters determined for some stars, similarly to what has been found for the impact of the activity in stellar structure of low mass stars (Morales et al. 2010). **Curiously, the three stars with low $v\ sin\ i < 6\ km/s$ (more precisely $\alpha CenA$, $\alpha CenB$, and the Sun) are also the oldest ones in our sample. Younger stars exhibit, in general, higher magnetic activity (Barnes 2003), and this can be mimicked by a lower mixing length parameter**.

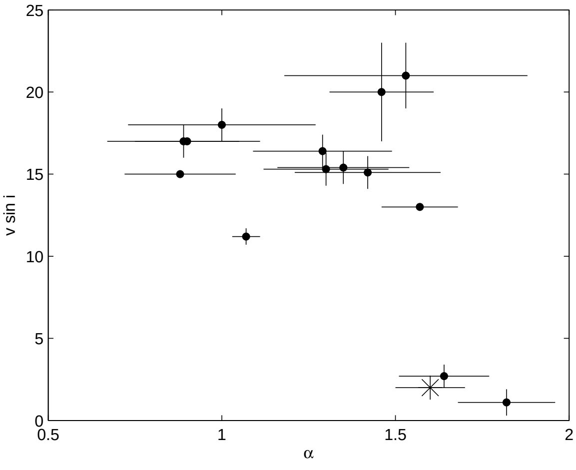

Figure 2. Mixing length parameter *vs* v sin i.

Some authors tried to establish direct relationships between the mixing length parameter and fundamental stellar properties, mainly the mass (e.g., Lastennet et al. 2003; Yildiz et al. 2006). In Figure 3, we plot the mass *versus* the mixing length parameter (once again the *Sun* is marked with a star). Even given the large dispersion, it seems that we are in the presence of an α decreasing with the increase of the mass. This is in agreement with the results coming from convection simulations (Ludwig et al. 1999; Trampedach & Stein 2011) but in disagreement with Lebreton et al. (2001) and Yildiz et al. (2006), who claimed that the main sequence of Hyades is better represented by the theoretical models, taking into account a variation of the mixing length parameter (α decreasing with the decrease of the mass). However, the comparison of our results to those ones must be made with caution. In fact, our set is a “mixing” of stars of different chemical compositions, in opposition to Lebreton et al. (2001) and Yildiz et al. (2006) who, working with Hyades members, considered a homogeneous group of stars. In other words, in Figure 3, we cannot disentangle the different contributions of the mass, the age, and the chemical composition.

Regarding the overshooting, we plot in Figure 4 its dependence on the mass (for the stars considered in this paper). Globally, it seems that the overshooting increases with an increase of the mass, as claimed by Ribas et al. (2000) (in our case the increase seems steeper). However, the error bars are large and we cannot exclude a constant value around 0.20 - 0.25 H_p for all stars, as pointed out by Claret (2007).

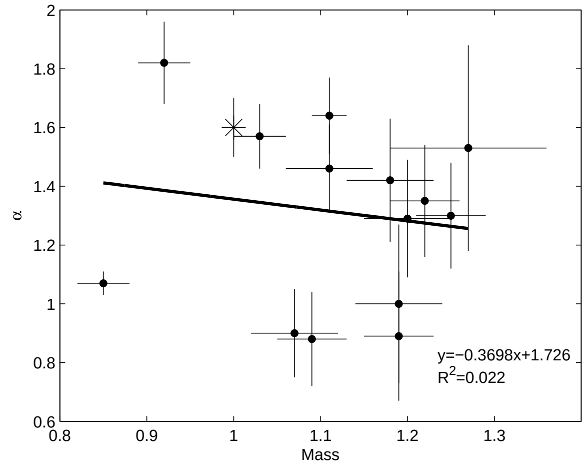

Figure 3. Mass *vs* mixing length parameter.

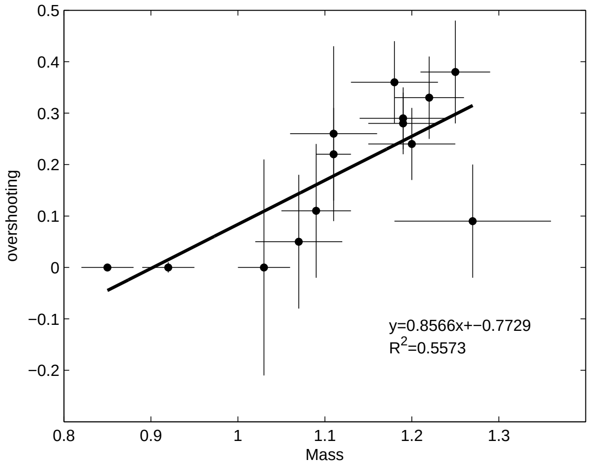

Figure 4. Mass *vs* overshooting.

4 CONCLUSIONS

We estimated the mass, the age, the helium and metals abundance, the convection parameters, and well as the corresponding estimation errors for the individual components of seven binary systems, where the members are all FGK main-sequence stars of Population I. One of our goals was to test if solar scaled models could be used for other stars (in particular in what regards the mixing length parameter and helium abundance in relation to metals). We came to the conclusion that for only two of these systems (VZ Hya and α *Cen*) the estimations of both components were compatible to solar models.

A global analysis of the fourteen stars yielded a mixing length parameter decreasing with the increase of the mass, but within some level of dispersion. We also realized that for $v \; sin \; i > 10 \; km/s$ the convection parameter of the stars was lower than the solar value, which has also also been supported by previous works. Such a tendency can be related to rotation. In fact, our models do not take into account the rotation and this effect could be mimicked by a decreasing mixing length parameter. Such a result deserves further investigation. On the other hand, our results support the claim of an overshooting increasing with the increase of the mass. For the helium-to-metal chemical enrichment parameter, solar values seem to be appropriated for all binaries. The extreme cases are WZ Oph (1.5) and NGC188 KR V12 (5.0), but even those can be considered (within our estimation errors) compatible to the solar value. Finally we point out that our results (with the exception of NGC188 KR V12) are in agreement to what has been obtained or predicted by previous works.

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