Bilevel and Multilevel Programming: A Bibliography Review[1]

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Abstract.
This paper contains a bibliography of all references central to bilevel and multilevel programming that the authors know of. It should be regarded as a dynamic and permanent contribution since, all the new and appropriate references that are brought to our attention will be periodically added to this bibliography. Readers are invited to suggest such additions, as well as corrections or modifications, and to obtain a copy of the LaTeX and BibTeX files that constitute this manuscript, using the guidelines contained in this paper.

To classify some of the references in this bibliography a short overview of past and current research in bilevel and multilevel programming is included. For those who are interested in but unfamiliar with the references in this area, we hope that this bibliography facilitates and encourages their research.

Keywords. Bilevel (two level), three level and multilevel programming, static Stackelberg problems, hierarchical optimization, minimax problems.

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1 Introduction and historical notes

Multilevel optimization problems are mathematical programs which have a subset of their variables constrained to be an optimal solution of other programs parameterized by their remaining variables. When these other programs are pure mathematical programs we are dealing with bilevel programming. Three level programming results when these other programs are themselves bilevel programs. By extending this idea it is possible to define multilevel programs with any number of levels.

The (continuous) bilevel programming problem (BPP) is defined as:

$$\begin{array}{ll} \min\limits_{x,y} & F(x,y) \\ \text{subject to} & g(x,y) \leq 0, \end{array}$$

where y, for each value of x, is the solution of the so–called lower level problem:

$$\begin{array}{ll} \min\limits_{y} & f(x,y) \\ \text{subject to} & h(x,y) \leq 0, \end{array}$$

with $x \in \mathbb{R}^{nx}$, $y \in \mathbb{R}^{ny}$, F, $f : \mathbb{R}^{nx+ny} \rightarrow \mathbb{R}$, $g : \mathbb{R}^{nx+ny} \rightarrow \mathbb{R}^{nu}$ and $h : \mathbb{R}^{nx+ny} \rightarrow \mathbb{R}^{nl}$. Variables x (y) are called the upper (respectively lower) level variables, $g(x,y) \leq 0$ ($h(x,y) \leq 0$) the upper (lower) level constraints and $F(x,y)$ ($f(x,y)$) the upper (lower) level objective function. Furthermore the relaxed problem associated with BPP can be stated as:

$$\begin{array}{ll} \min\limits_{x,y} & F(x,y) \\ \text{subject to} & g(x,y) \leq 0, \quad h(x,y) \leq 0, \end{array}$$

and its optimal value is a lower bound for the optimal value of the BPP. Other important BPP definitions and notations are itemized below.

- the relaxed feasible set (or constraint region),

$$\Omega = \{(x,y) : \ g(x,y) \leq 0, \ h(x,y) \leq 0\}.$$

- for each x, the lower level feasible set,

$$\Omega(x) = \{y : h(x,y) \leq 0\}.$$

- for each x, the lower level reaction set (follower's feasible region),

$$M(x) = \{y : y \in argmin\{f(x,y) : \ y \in \Omega(x)\}\}.$$

- for each x and any value of y in $M(x)$, the lower level optimal value,

$$v(x) = f(x,y)$$

- the induced (inducible) region,

$$IR = \{(x, y) : \ (x, y) \in \Omega, \ y \in M(x)\}.$$

The induced region is the feasible set of the BPP. It is usually nonconvex and, in the presence of upper level constraints, can be disconnected or even empty. The reader is referred to T. Edmunds and J. Bard [73] for a short description of the conditions under which the induced region is compact and the BPP has an optimal solution.

The BPP is convex if $f(x, y)$ and $h(x, y)$ are convex functions in y for all values of x (i.e., if the lower level problem is convex). The convex BPP has received most of the attention in the literature. The advantage of dealing with the convex BPP is that under an appropriate constraint qualification, the lower level problem can be replaced by its Karush–Kuhn–Tucker (KKT) conditions to obtain an equivalent (one–level) mathematical program. However, despite their designation, convex BPPs have nonconvex induced regions that can be disconnected or even empty in the presence of upper level constraints. There are three important classes of convex BPPs, namely:

- the linear BPP, where all functions involved are affine.
- the linear–quadratic BPP, where the lower level objective is a convex quadratic and all remaining functions are affine.
- the quadratic BPP, that differs from the linear–quadratic BPP in that the upper level objective is also a quadratic function.

The original formulation for bilevel programming appeared in 1973, in a paper authored by J. Bracken and J. McGill [41], although is was W. Candler and R. Norton [51] that first used the designation *bilevel* and *multilevel programming*. However, it was not until the early eighties that these problems started receiving the attention they deserve. Motivated by the game theory of H. Stackelberg [150], several authors studied bilevel programming intensively and contributed to its proliferation in the mathematical programming community. At this stage, references such as E. Aiyoshi and K. Shimizu [1], [146], [147], J. Bard and J. Falk [11], [20], W. Bialas, H. Karwan and J. Shaw [34], [35], [37], W. Candler, J. Fortuny-Amat, B. McCarl, R. Norton and R. Townley [50], [51], [52], [53], [77] and U. Wen [163] should be distinguished.

Since 1980 a significant effort has been devoted to understanding the fundamental concepts associated with bilevel programs. At the same time several algorithms have been proposed for solving these problems. Important surveys of these efforts include those by C. Kolstad [99], G. Savard [140] and G. Anandalingam and T. Friesz [8]. Recently, a survey on the linear case has been written by O. Ben-Ayed [25].

2 Properties of bilevel programs

It is our opinion that bilevel programming represents an interesting and rich field of mathematical programming and although some important results have already been obtained it is still a fertile area for research. In this section we list some of the well–known properties of the BPP.

Optimality conditions

Several different optimality conditions have been proposed in the literature.

A first attempt was made by J. Bard [16] using an equivalence with a one–level mathematical program having an infinite and parametric set of constraints. However a counter example to these conditions was discovered by P. Clarke and A. Westerberg [61] and by A. Haurie, G. Savard and D. White [85]. Consequently, two algorithms based on these conditions (proposed in [13], [14] and [158]) are not convergent (see [140]).

Y. Chen and M. Florian [57], S. Dempe [66, 67], Y. Ishizuka [88], J. Outrata [135], and J. Ye and D. Zhu [174] used nonsmooth analysis, whereas Z. Bi and P. Calamai [31] explored the relationship between the BPP and an associated exact penalty function, to derive other necessary and sufficient optimality conditions.

Unlike much of the optimality analysis that has been done for (one–level) mathematical programs these aforementioned contributions have mostly ignored the special geometry of the BPP. To partially address this void G. Savard and J. Gauvin [141] have proposed necessary optimality conditions based on the concept of the steepest descent direction. More directly, L. Vicente and P. Calamai [160] have proposed necessary and sufficient optimality conditions, based on the geometry of the BPP, that are generalizations of the well–known first and second order optimality conditions for mathematical programs.

Complexity

The difficulty and complexity of the BPP is easily confirmed by looking at what might be considered its simplest version, the linear BPP. Examples of linear BPPs with an exponential number of local minima can be generated using a technique proposed by P. Calamai and L. Vicente [48]. R. Jeroslow [92] showed that the linear BPP is NP–Hard. A few years later, J. Bard [19] and O. Ben-Ayed and C. Blair [26], confirmed this result and presented shorter proves. The tightest complexity result is due to P. Hansen, B. Jaumard and G. Savard [81], where it is established that the linear BPP is strongly NP–Hard. Recently, L. Vicente, G. Savard and J. Júdice [162] have shown that checking local optimality in a linear BPP is a NP–Hard problem.

Related problems

The fact that important mathematical programs, such as minimax problems, linear integer, bilinear and quadratic programs, can be stated as special instances of bilevel programs illustrates the importance of these problems.

Although it is a simple matter to see that a minimax problem can be rewritten as a BPP problem, the first authors exploiting the reduction of a bilinear program to a linear BPP were G. Gallo and A. Ülkücü [80]. This result also established that any integer or concave quadratic program could be written as a linear BPP. One might think that any linear BPP can also be reduced to a bilinear program, thereby establishing an equivalence between these problems. However this is not entirely possible since the reciprocal result states that there exists a (penalized) bilinear program whose optimal (global) solutions are also global solutions of the corresponding linear BPP (see [171]). Finally, the reduction of any quadratic program to a quadratic BPP with bilinear objective functions is described in L. Vicente [159].

Although several authors have attempted to establish a link between two objective optimization and bilevel programming (J. Bard [16] and G. Ünlü [158]), none have succeeded thus far in proposing conditions that guarantee that the optimal solution of a given bilevel program is Pareto-optimal or efficient [85] for both upper and lower level objective functions (W. Candler [49], P. Clarke and A. Westerberg [61], A. Haurie, G. Savard and D. White [85], P. Marcotte [115], P. Marcotte and G. Savard [117] and U. Wen and S. Hsu [167]).

The static Stackelberg problem (SSP) can be posed as:

$$\begin{aligned} \min_{x} \quad & F(x,y) \\ \text{subject to} \quad & g(x,y) \leq 0, \\ & y \in argmin\{f(x,y):\ h(x,y) \leq 0\}, \end{aligned}$$

and differs from the BPP in the way the upper level function is minimized. If the reaction set $\{y : y \in argmin\{f(x,y) : h(x,y) \leq 0\}\}$ is not a singleton for some values of x with $g(x,y) \leq 0$, then a solution of the SSP might not be a solution of the BPP. Comments on this problem and its relationships with game theory can be found in [140].

Other two–level optimization problems might also be confused with bilevel programs. That is the case with the following problem studied by T. Tanino and T. Ogawa [154].

$$\begin{aligned} \min_{x,y} \quad & \mathcal{F}(x) = F(x, v(x)) \\ \text{subject to} \quad & \mathcal{G}(x) \leq 0, \end{aligned}$$

where y, for each value of x, is the solution of the second optimization problem:

$$\begin{aligned} \min_{y} \quad & f(x,y) \\ \text{subject to} \quad & h(x,y) \leq 0, \end{aligned}$$

$v(x)$ is the optimal value of the second problem parameterized by x, and $\mathcal{G} : \mathbb{R}^{nx} \rightarrow \mathbb{R}^{nu}$. Under certain convexity and differentiability assumptions these authors have demonstrated that this problem can be treated as a one–level convex optimization problem and proposed a descent algorithm for its solution.

Authors who have studied *generalized* bilevel programming problems include T. Friesz et. al. [79], J. Outrata [136] and P. Marcotte and D. Zhu [119] who replaced the BPP lower level problem with a variational inequality problem.

3 Solution of bilevel programs

The algorithms that have been proposed for solving continuous bilevel programming problems may be divided in five different classes. In most cases these algorithms can be tested and compared using the test problem generators proposed by P. Calamai and L. Vicente [46], [48], [47] for generating linear, linear–quadratic and quadratic BPPs.

Extreme point algorithms

Most of these algorithms are applied to the solution of linear BPPs. Every linear BPP with a finite optimal solution shares the important property that at least one optimal (global) solution is attained at an extreme point of the set Ω. This result was first established by W. Candler and R. Townsley [53] for linear BPPs with no upper level constraints and with unique lower level solutions. Afterwards J. Bard [15] and W. Bialas and M. Karwan [36] proved it under the assumption that Ω is bounded. The result for the case where upper level constraints exist has been established by G. Savard [140] under no particular assumptions. We remark that this property is no longer valid for linear–quadratic BPPs.

Based on this property, W. Candler and R. Townsley [53] and W. Bialas and M. Karwan [36] have proposed algorithms that compute global solutions of linear BPPs by enumerating the extreme points of Ω. Whereas the former algorithm enumerates basis of the lower level problem, the latter, known as the "Kth–best", enumerates basis of the relaxed problem. Other extreme point approaches for linear BPPs have been proposed by Y. Chen and M. Florian [58], [60], S. Dempe [64], G. Papavassilopoulos [138] and H. Tuy, A. Midgalas and P. Värbrand [156].

L. Vicente, G. Savard and J. Júdice [162] have studied the induced regions of the quadratic BPP and introduced the concepts of *extreme induced region points* and *extreme induced region directions*. They proposed extreme point algorithms that compute local star minima and local minima depending on the nature of the upper level objective function.

Branch and bound algorithms

Branch and bound methods are widely applied to convex bilevel programs. Although they are associated with large computational efforts they are also capable of computing global minima. Several approaches exploit the complementarity between the multipliers and the slack variables that arises from the KKT conditions of the lower level problem. That is the case of the algorithms proposed by J. Bard and J. Falk [20] and J. Fortuny-Amat and B. McCarl [77] for the linear case, J. Bard and J. Moore [21] for the linear–quadratic case and F. Al-Khayyal, R. Horst and P. Pardalos [3], J. Bard [18] and T. Edmunds and J. Bard [73] for the quadratic case. Using different branching strategies, P. Hansen, B. Jaumard and G. Savard [81] have proposed a branch and bound algorithm for the solution of the linear BPP that seems particularly efficient for the solution of medium–scale problems.

Although little attention has been given to the case in which some variables are restricted to have integer values J. Bard and J. Moore [22], [124] and U. Wen and Y. Yang [170] have proposed branch and bound procedures for the solution of integer linear instances of the BPP, and T. Edmunds and J. Bard [74] have introduced a branch and bound algorithm for the solution of the integer quadratic BPP.

Complementarity pivot algorithms

The first complementarity pivot algorithm for solving linear BPPs was proposed by W. Bialas, M. Karwan and J. Shaw [37]. This algorithm cannot, as suggested in [36], compute global solutions of linear BPPs (see examples in [26] and [93]).

By combining some of the ideas from the last two classes of algorithms, J. Júdice and A. Faustino proposed the SLCP (sequential linear complementarity problem) algorithm for the computation of ϵ–global solutions of linear ([93], [94]) and linear–quadratic ([95]) BPPs. This algorithm seems quite efficient for the solution of medium–scale problems.

Another complementarity pivot approach which can be classified as a modified simplex approach was proposed by H. Önal [133].

Descent methods

In this class we include methods incorporating descent directions that are designed to compute stationary points and local minima. A classical example is the steepest descent direction algorithm extended to nonlinear bilevel programming by G. Savard and J. Gauvin [141]. Here the computation of the steepest descent direction for a BPP is done with the help of a linear–quadratic BPP. L. Vicente, G. Savard and J. Júdice [162] studied the application of this algorithm to convex bilevel programming, where the lower level problems are strictly convex quadratic programs, and proposed appropriate stepsize rules to displacements along directions in the induced region.

A second classical algorithm is the one proposed by C. Kolstad and L. Lasdon [100] for the solution of nonlinear BPPs. This algorithm consists of applying gradient information to the implicit optimization problem:

$$\begin{array}{ll} \min\limits_{x} & F(x, y(x)) \\ \text{subject to} & g(x, y(x)) \leq 0 \end{array}$$

where $\{y(x)\}$ is the lower level reaction set for all values of x. The authors introduced a local estimation of the gradient of y and applied a BFGS quasi–Newton algorithm to the solution of an unconstrained version of this problem.

Another descent approach can be found in M. Florian and Y. Chen [75].

Penalty function methods

Some of the methods in this class can also be classified as descent algorithms. They usually incorporate exact penalty functions and are limited to computing stationary points and local minima. See E. Aiyoshi and K. Shimizu [1], [2], [147], and Z. Bi, P. Calamai and A.R. Conn [32], [33] for the case where the penalty term incorporates the lower level objective function, and Y. Ishizuka and E. Aiyoshi [89] for the case where both objective functions are penalized. The reader is also referred to the work of P. Loridan and J. Morgan on approximation and stability results for bilevel programming that might be of interest for the convergence theory of these and other algorithms, and to Z.-Q. Luo, J.-S. Pang and S. Wu [112] for the derivation of an exact penalty function that only uses the square-root of the complementarity term associated with the lower level quadratic program as the penalty term.

In [171], D. White and G. Anandalingam exploit the penalized bilinear version of a linear BPP and introduce a exact penalty function algorithm that finds a global solution of the linear BPP by solving a sequence of bilinear programs.

4 Multilevel programming and applications

As stated before, bilevel programming is a special case of multilevel programming. However, as described by C. Blair [39], the complexity of these problems increases significantly when the number of levels is greater than two. In spite of this, three level and multilevel programming has been studied in the literature by, among others, J. Bard [15], J. Bard and J. Falk [20], H. Benson [29], R. Jan and M. Chen [90] and U. Wen and W. Bialas [166].

The particular structure of bilevel and multilevel programs facilitates the formulation of a number of practical problems that involve an hierarchical decision making process. Among the several applications of bilevel and multilevel programming the following are noteworthy:

- **Transportation** – Network design problem (L. LeBlanc and D. Boyce [102], O. Ben-Ayed, C. Blair, D. Boyce and L. LeBlanc [27], [28], P. Marcotte [114], P. Marcotte and G. Marquis [116] and S. Suh and T. Kim [151]) and trip demand estimation problem (M. Florian and Y. Chen [75], [76] and R.L. Tobin and T.L. Friesz [155]).

- **Management** – Coordination of multidivisional firms (J. Bard [13]), network facility location with delivered price competition (T. Miller, T. Friesz and R. Tobin [122]) and credit allocation (R. Cassidy and M. Kirby and W. Raike [54]).

- **Planning** – Application of agricultural policies (W. Candler, J. Fortuny-Amat and B. McCarl [50], W. Candler and R. Norton [51], [52] and H. Önal [132]) and electric utility planning (A. Haurie, R. Loulou and G. Savard [83] and B. Hobbs and S. Nelson [86]).

- **Engineering Design** – Optimal design problems (M. Kocvara and J. Outrata [97], [98] and P. Neittaanmäki and A. Stachurski [130]).

We believe that bilevel programming can play an important role in other branches of mathematical programming. For example, bilevel programming can provide a novel approach for analyzing the step selection subproblem in a trust region algorithm for nonlinear equality constrained optimization (see [55]), and has been applied to the discriminant problem [118].

5 How to contribute and how to get this report

The subjects covered in this bibliography review are bilevel and multilevel programming and Stackelberg problems when considered as optimization problems – usually called static Stackelberg problems. We have selected contributions in this area that deal with theory issues (properties, existence of solution, optimality conditions and so on), algorithms and numerical results, software and generation of test problems, applications and complexity issues.

References to be cited should be books, articles published in journals or special volumes and technical reports that are available to the broad research community. Conferences and seminar abstracts are not included.

Many of the references listed in our bibliography have been cited in the text however for completeness we have included all qualifying references that we are familiar with.

This bibliography review is updated biannually and is available via email or anonymous ftp. It consists of the BibTeX file bilevel-review.bib that contains the bibliographic entries and the LaTeX file bilevel-review.tex that constitutes this manuscript. In order to get these files:

- Most preferably, using anonymous ftp:

  Compressed versions of these two files can be obtained using the procedure described below. Entries on the left are the prompts (typewriter type style) and example responses (bold typewriter type style) whereas those on the right are comments that describe the corresponding action.

`%` **ftp dial.uwaterloo.ca**	connect to machine
`Name (machine:userid):` **anonymous**	use userid anonymous
`Password:` **jqpublic@domain**	use email address password
`ftp>` **cd pub/phcalamai/bilevel-review**	move to correct directory
`ftp>` **binary**	set transfer type
`ftp>` **get bilevel-review.tex.Z**	fetch compressed latex file
`ftp>` **get bilevel-review.bib.Z**	fetch compressed bib file
`ftp>` **quit**	terminate ftp session
`%` **uncompress bilevel-review.tex.Z**	expand latex file
`%` **uncompress bilevel-review.bib.Z**	expand bib file
`%` **latex bilevel-review**	create aux file
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All contributions, corrections and suggestions are welcome and should be sent to either of the authors' addresses or (preferably) to the email address listed above.

References

[1] E. Aiyoshi and K. Shimizu. Hierarchical decentralized systems and its new solution by a barrier method. *IEEE Transactions on Systems, Man, and Cybernetics*, 11:444–449, 1981.

[2] E. Aiyoshi and K. Shimizu. A solution method for the static constrained Stackelberg problem via penalty method. *IEEE Transactions on Automatic Control*, 29:1111–1114, 1984.

[3] F. Al-Khayyal, R. Horst, and P. Pardalos. Global optimization of concave functions subject to quadratic constraints: an application in nonlinear bilevel programming. *Annals of Operations Research*, 34:125–147, 1992.

[4] N. Alexandrov and J. E. Dennis. Algorithms for bilevel optimization. Technical Report TR94–34, Department of Computational and Applied Mathematics, Rice University, 1994.

[5] G. Anandalingam. An analysis of information and incentives in bi-level programming. In *IEEE 1985 Proceedings of the International Conference on Cybernetics and Society*, pages 925–929, 1985.

[6] G. Anandalingam. A mathematical programming model of decentralized multi-level systems. *Journal of the Operational Research Society*, 39:1021–1033, 1988.

[7] G. Anandalingam and V. Aprey. Multi-level programming and conflict resolution. *European Journal of Operational Research*, 51:233–247, 1991.

[8] G. Anandalingam and T. Friesz. Hierarchical optimization: an introduction. *Annals of Operations Research*, 34:1–11, 1992.

[9] G. Anandalingam, R. Mathieu, L. Pittard, and N. Sinha. Artificial intelligence based approaches for solving hierarchical optimization problems. In R. Sharda, B. Golden, E. Wasil, O. Balci and W. Stewart, editor, *Impacts of Recent Computer Advances on Operations Research*, pages 289–301. Elsevier Science Publishing Co., Inc., 1983.

[10] G. Anandalingam and D. White. A solution method for the linear static Stackelberg problem using penalty functions. *IEEE Transactions on Automatic Control*, 35:1170–1173, 1990.

[11] J. Bard. A grid search algorithm for the linear bilevel programming problem. In *Proceedings of the 14th Annual Meeting of the American Institute for Decision Science*, pages 256–258, 1982.

[12] J. Bard. An algorithm for solving the general bilevel programming problem. *Mathematics of Operations Research*, 8:260–272, 1983.

[13] J. Bard. Coordination of a multidivisional organization through two levels of management. *OMEGA*, 11:457–468, 1983.

[14] J. Bard. An efficient point algorithm for a linear two-stage optimization problem. *Operations Research*, 31:670–684, 1983.

[15] J. Bard. An investigation of the linear three level programming problem. *IEEE Transactions on Systems, Man, and Cybernetics*, 14:711–717, 1984.

[16] J. Bard. Optimality conditions for the bilevel programming problem. *Naval Research Logistics Quarterly*, 31:13–26, 1984.

[17] J. Bard. Geometric and algorithm developments for a hierarchical planning problem. *European Journal of Operational Research*, 19:372–383, 1985.

[18] J. Bard. Convex two-level optimization. *Mathematical Programming*, 40:15–27, 1988.

[19] J. Bard. Some properties of the bilevel programming problem. *Journal of Optimization Theory and Applications*, 68:371–378, 1991. Technical Note.

[20] J. Bard and J. Falk. An explicit solution to the multi-level programming problem. *Computers and Operations Research*, 9:77–100, 1982.

[21] J. Bard and J. Moore. A branch and bound algorithm for the bilevel programming problem. *SIAM Journal on Scientific and Statistical Computing*, 11:281–292, 1990.

[22] J. Bard and J. Moore. An algorithm for the discrete bilevel programming problem. *Naval Research Logistics*, 39:419–435, 1992.

[23] O. Ben-Ayed. *Bilevel linear programming: analysis and application to the network design problem.* PhD thesis, University of Illinois at Urbana-Champaign, 1988.

[24] O. Ben-Ayed. A bilevel linear programming model applied to the Tunisian inter-regional network design problem. *Revue Tunisienne d'Économie et de Gestion*, 5:235–279, 1990.

[25] O. Ben-Ayed. Bilevel linear programming. *Computers and Operations Research*, 20:485–501, 1993.

[26] O. Ben-Ayed and C. Blair. Computational difficulties of bilevel linear programming. *Operations Research*, 38:556–560, 1990.

[27] O. Ben-Ayed, C. Blair, D. Boyce, and L. LeBlanc. Construction of a real-world bilevel linear programming model of the highway design problem. *Annals of Operations Research*, 34:219–254, 1992.

[28] O. Ben-Ayed, D. Boyce, and C. Blair. A general bilevel linear programming formulation of the network design problem. *Transportation Research*, 22 B:311–318, 1988.

[29] H. Benson. On the structure and properties of a linear multilevel programming problem. *Journal of Optimization Theory and Applications*, 60:353–373, 1989.

[30] Z. Bi. *Numerical methods for bilevel programming problems.* PhD thesis, Department of Systems Design Engineering, University of Waterloo, 1992.

[31] Z. Bi and P. Calamai. Optimality conditions for a class of bilevel programming problems. Technical Report #191-O-191291, Department of Systems Design Engineering, University of Waterloo, 1991.

[32] Z. Bi, P. Calamai, and A. Conn. An exact penalty function approach for the linear bilevel programming problem. Technical Report #167-O-310789, Department of Systems Design Engineering, University of Waterloo, 1989.

[33] Z. Bi, P. Calamai, and A. Conn. An exact penalty function approach for the nonlinear bilevel programming problem. Technical Report #180-O-170591, Department of Systems Design Engineering, University of Waterloo, 1991.

[34] W. Bialas and M. Karwan. Multilevel linear programming. Technical Report 78-1, Operations Research Program, State University of New York at Buffalo, 1978.

[35] W. Bialas and M. Karwan. On two-level optimization. *IEEE Transactions on Automatic Control*, 27:211–214, 1982.

[36] W. Bialas and M. Karwan. Two-level linear programming. *Management Science*, 30:1004–1020, 1984.

[37] W. Bialas, M. Karwan, and J. Shaw. A parametric complementary pivot approach for two-level linear programming. Technical Report 80-2, Operations Research Program, State University of New York at Buffalo, 1980.

[38] J. Bisschop, W. Candler, J. Duloy, and G. O'Mara. The indus basin model: a special application of two-level linear programming. *Mathematical Programming Study*, 20:30–38, 1982.

[39] C. Blair. The computational complexity of multi-level linear programs. *Annals of Operations Research*, 34:13–19, 1992.

[40] J. Bracken, J. Falk, and J. McGill. Equivalence of two mathematical programs with optimization problems in the constraints. *Operations Research*, 22:1102–1104, 1974.

[41] J. Bracken and J. McGill. Mathematical programs with optimization problems in the constraints. *Operations Research*, 21:37–44, 1973.

[42] J. Bracken and J. McGill. Defense applications of mathematical programs with optimization problems in the constraints. *Operations Research*, 22:1086–1096, 1974.

[43] J. Bracken and J. McGill. A method for solving mathematical programs with nonlinear programs in the constraints. *Operations Research*, 22:1097–1101, 1974.

[44] J. Bracken and J. McGill. Production and marketing decisions with multiple objectives in a competitive environment. *Journal of Optimization Theory and Applications*, 24:449–458, 1978.

[45] P. Calamai and L. Vicente. Generating linear and linear-quadratic bilevel programming problems. *SIAM Journal on Scientific and Statistical Computing*, 14:770–782, 1993.

[46] P. Calamai and L. Vicente. Algorithm 728: Fortran subroutines for generating quadratic bilevel programming problems. *Collected Algorithms from ACM*, 728-P:1.0–3.0, 1994. (Code published in ACM Supplement xxx:1–?, 1994).

[47] P. Calamai and L. Vicente. Algorithm 728: Fortran subroutines for generating quadratic bilevel programming problems. *ACM Transactions on Mathematical Software*, 20:120–123, 1994.

[48] P. Calamai and L. Vicente. Generating quadratic bilevel programming problems. *ACM Transactions on Mathematical Software*, 20:103–119, 1994.

[49] W. Candler. A linear bilevel programming algorithm: A comment. *Computers and Operations Research*, 15:297–298, 1988.

[50] W. Candler, J. Fortuny-Amat, and B. McCarl. The potential role of multilevel programming in agricultural economics. *American Journal of Agricultural Economics*, 63:521–531, 1981.

[51] W. Candler and R. Norton. Multilevel programming. Technical Report 20, World Bank Development Research Center, Washington D.C., 1977.

[52] W. Candler and R. Norton. Multilevel programming and development policy. Technical Report 258, World Bank Staff, Washington D.C., 1977.

[53] W. Candler and R. Townsley. A linear two-level programming problem. *Computers and Operations Research*, 9:59–76, 1982.

[54] R. Cassidy, M. Kirby, and W. Raike. Efficient distribution of resources through three levels of government. *Management Science*, 17:462–473, 1971.

[55] M. Cellis, J. Dennis, and R. Tapia. A trust region strategy for nonlinear equality constrained optimization. In *Numerical Optimization 1984*, Proceedings 20, pages 71–82. SIAM, Philadelphia, 1985.

[56] Y. Chen. *Bilevel programming problems: analysis, algorithms and applications.* PhD thesis, Université de Montréal, 1993.

[57] Y. Chen and M. Florian. The nonlinear bilevel programming problem: a general formulation and optimality conditions. Technical Report CRT-794, Centre de Recherche sur les Transports, 1991.

[58] Y. Chen and M. Florian. On the geometry structure of linear bilevel programs: a dual approach. Technical Report CRT-867, Centre de Recherche sur les Transports, 1992.

[59] Y. Chen and M. Florian. The nonlinear bilevel programming problem: formulations, regularity and optimality conditions. Technical Report CRT-794, Centre de Recherche sur les Transports, 1993.

[60] Y. Chen, M. Florian, and S. Wu. A descent dual approach for linear bilevel programs. Technical Report CRT-866, Centre de Recherche sur les Transports, 1992.

[61] P. Clarke and A. Westerberg. A note on the optimality conditions for the bilevel programming problem. *Naval Research Logistics*, 35:413–418, 1988.

[62] P. Clarke and A. Westerberg. Bilevel programming for steady-state chemical process design-I. Fundamentals and algorithms. *Computers & Chemical Engineering*, 14:87–98, 1990.

[63] P. Clarke and A. Westerberg. Bilevel programming for steady-state chemical process design-II. Performance study for nondegenerate problems. *Computers & Chemical Engineering*, 14:99–110, 1990.

[64] S. Dempe. A simple algorithm for the linear bilevel programming problem. *Optimization*, 18:373–385, 1987.

[65] S. Dempe. On one optimality condition for bilevel optimization. *Vestnik Leningrad Gos. University*, pages 10–14, 1989. Serija I, in Russian, translation Vestnik Leningrad University, Math., 22:11–16, 1989.

[66] S. Dempe. A necessary and a sufficient optimality condition for bilevel programming problems. *Optimization*, 25:341–354, 1992.

[67] S. Dempe. Optimality conditions for bilevel programming problems. In P. Kall, editor, *System modelling and optimization*, pages 17–24. Springer-Verlag, 1992.

[68] A. deSilva. *Sensitivity formulas for nonlinear factorable programming and their application to the solution of an implicitly defined optimization model of US crude oil production*. PhD thesis, George Washington University, 1978.

[69] A. deSilva and G. McCormick. Implicitly defined optimization problems. *Annals of Operations Research*, 34:107–124, 1992.

[70] Y. Dirickx and L. Jennegren. *Systems analysis by multi-level methods: with applications to economics and management*. John Wiley, New York, 1979.

[71] O. Drissi-Kaitouni and J.T. Lundgren. Bilevel origin-destination matrix estimation using a descent approach. Technical Report LiTH-MAT-R-1992-49, Department of Mathematics, Linkoping Institute of Technology, Sweden, 1992.

[72] T. Edmunds. *Algorithms for nonlinear bilevel mathematical programs*. PhD thesis, Department of Mechanical Engineering, University of Texas at Austin, 1988.

[73] T. Edmunds and J. Bard. Algorithms for nonlinear bilevel mathematical programming. *IEEE Transactions on Systems, Man, and Cybernetics*, 21:83–89, 1991.

[74] T. Edmunds and J. Bard. An algorithm for the mixed-integer nonlinear bilevel programming problem. *Annals of Operations Research*, 34:149–162, 1992.

[75] M. Florian and Y. Chen. A bilevel programming approach to estimating O-D matrix by traffic counts. Technical Report CRT-750, Centre de Recherche sur les Transports, 1991.

[76] M. Florian and Y. Chen. A coordinate descent method for bilevel O-D matrix estimation problems. Technical Report CRT-807, Centre de Recherche sur les Transports, 1993.

[77] J. Fortuny-Amat and B. McCarl. A representation and economic interpretation of a two-level programming problem. *Journal of the Operational Research Society*, 32:783–792, 1981.

[78] T. Friesz, C. Suwansirikul, and R. Tobin. Equilibrium decomposition optimization: a heuristic for the continuous equilibrium network design problem. *Transportation Science*, 21:254–263, 1987.

[79] T. Friesz, R. Tobin, H. Cho, and N. Mehta. Sensitivity analysis based heuristic algorithms for mathematical programs with variational inequality constraints. *Mathematical Programming*, 48:265–284, 1990.

[80] G. Gallo and A. Ülkücü. Bilinear programming: an exact algorithm. *Mathematical Programming*, 12:173–194, 1977.

[81] P. Hansen, B. Jaumard, and G. Savard. New branch-and-bound rules for linear bilevel programming. *SIAM Journal on Scientific and Statistical Computing*, 13:1194–1217, 1992.

[82] P. Harker and J.-S. Pang. Existence of optimal solutions to mathematical programs with equilibrium constraints. *Operations Research Letters*, 7:61–64, 1988.

[83] A. Haurie, R. Loulou, and G. Savard. A two-level systems analysis model of power cogeneration under asymmetric pricing. In *Proceedings of IEEE Automatic Control Conference*, San Diego, May 1990.

[84] A. Haurie, R. Loulou, and G. Savard. A two player game model of power cogeneration in new england. *IEEE Transactions on Automatic Control*, 37:1451–1456, 1992.

[85] A. Haurie, G. Savard, and D. White. A note on: an efficient point algorithm for a linear two-stage optimization problem. *Operations Research*, 38:553–555, 1990.

[86] B. Hobbs and S. Nelson. A nonlinear bilevel model for analysis of electric utility demand-side planning issues. *Annals of Operations Research*, 34:255–274, 1992.

[87] S. Hsu and U. Wen. A review of linear bilevel programming problems. In *Proceedings of the National Science Council, Republic of China, Part A: Physical Science and Engineering*, volume 13, pages 53–61, 1989.

[88] Y. Ishizuka. Optimality conditions for quasi-differentiable programs with applications to two-level optimization. *SIAM Journal on Control and Optimization*, 26:1388–1398, 1988.

[89] Y. Ishizuka and E. Aiyoshi. Double penalty method for bilevel optimization problems. *Annals of Operations Research*, 34:73–88, 1992.

[90] R. Jan and M. Chern. Multi-level nonlinear integer programming. 1990. (Preprint from the Department of Computer and Information Science, National Chiao Tung University).

[91] R. Jan and M. Chern. Nonlinear integer bilevel programming. *European Journal of Operational Research*, 72:574–587, 1994.

[92] R. Jeroslow. The polynomial hierarchy and a simple model for competitive analysis. *Mathematical Programming*, 32:146–164, 1985.

[93] J. Júdice and A. Faustino. The solution of the linear bilevel programming problem by using the linear complementarity problem. *Investigação Operacional*, 8:77–95, 1988.

[94] J. Júdice and A. Faustino. A sequential LCP method for bilevel linear programming. *Annals of Operations Research*, 34:89–106, 1992.

[95] J. Júdice and A. Faustino. The linear-quadratic bilevel programming problem. *INFOR*, 32:87–98, 1994.

[96] T. Kim and S. Suh. Toward developing a national transportation planning model: a bilevel programming approach for Korea. *Annals of Regional Science*, 22:65–80, 1988.

[97] M. Kocvara and J. Outrata. A nondifferentiable approach to the solution of optimum design problems with variational inequalities. In *P. Kall, ed., System Modelling and Optimization*, Lecture Notes in Control and Information Sciences 180, pages 364–373. Springer-Verlag, Berlin, 1992.

[98] M. Kocvara and J. Outrata. A numerical solution of two selected shape optimization problems. Technical Report DFG (German Scientific Foundation) Research Report 464, University of Bayreuth, 1993.

[99] C. Kolstad. A review of the literature on bi-level mathematical programming. Technical Report LA-10284-MS, US-32, Los Alamos National Laboratory, 1985.

[100] C. Kolstad and L. Lasdon. Derivative evaluation and computational experience with large bilevel mathematical programs. *Journal of Optimization Theory and Applications*, 65:485–499, 1990.

[101] M. Labbé, P. Marcotte, and G. Savard. A bilevel model of taxation and its application to optimal highway policy. 1993. Preprint.

[102] L. Leblanc and D. Boyce. A bilevel programming algorithm for exact solution of the network design problem with user-optimal flows. *Transportation Research*, 20 B:259–265, 1986.

[103] Y. Liu and S. Hart. Characterizing an optimal solution to the linear bilevel programming problem. *European Journal of Operational Research*, 73:164–166, 1994.

[104] Y. Liu and T. Spencer. Solving a bilevel linear program when the inner decision maker controls few variables. *European Journal of Operational Research*, 81:644–651, 1995.

[105] P. Loridan and J. Morgan. Approximate solutions for two-level optimization problems. In K. Hoffman, J. Hiriart-Urruty, C. Lamerachal and J. Zowe, editor, *Trends in Mathematical Optimization*, volume 84 of *International Series of Numerical Mathematics*, pages 181–196. Birkhäuser Verlag, Basel, 1988.

[106] P. Loridan and J. Morgan. A theoretical approximation scheme for Stackelberg problems. *Journal of Optimization Theory and Applications*, 61:95–110, 1989.

[107] P. Loridan and J. Morgan. ϵ-Regularized two-level optimization problems: approximation and existence results. In *Optimization – Fifth French-German Conference*, Lecture Notes in Mathematics 1405, pages 99–113. Springer-Verlag, Berlin, 1989.

[108] P. Loridan and J. Morgan. New results on approximate solutions in two-level optimization. *Optimization*, 20:819–836, 1989.

[109] P. Loridan and J. Morgan. *Quasi convex lower level problem and applications in two level optimization*, volume 345 of *Lecture Notes in Economics and Mathematical Systems*, pages 325–341. Springer-Verlag, Berlin, 1990.

[110] P. Loridan and J. Morgan. A sequential stability result for constrained Stackelberg problems. *Richerche di Matematic*, (forthcoming).

[111] P. Luh, T.-S. Chang, and T. Ning. Three–level Stackelberg decision problems. *IEEE Transactions on Automatic Control*, 29:280–282, 1984.

[112] Z.-Q. Luo, J.-S. Pang, and S. Wu. Exact penalty functions for mathematical programs and bilevel programs with analytic constraints. 1993. Preprint from the Department of Electrical and Computer Engineering, McMaster University.

[113] P. Marcotte. Network optimization with continuous control parameters. *Transportation Science*, 17:181–197, 1983.

[114] P. Marcotte. Network design problem with congestion effects: a case of bilevel programming. *Mathematical Programming*, 34:142–162, 1986.

[115] P. Marcotte. A note on bilevel programming algorithm by LeBlanc and Boyce. *Transportation Research*, 22 B:233–237, 1988.

[116] P. Marcotte and G. Marquis. Efficient implementation of heuristics for the continuous network design problem. *Annals of Operations Research*, 34:163–176, 1992.

[117] P. Marcotte and G. Savard. A note on the pareto optimality of solutions to the linear bilevel programming problem. *Computers and Operations Research*, 18:355–359, 1991.

[118] P. Marcotte and G. Savard. Novel approaches to the discrimination problem. *ZOR – Methods and Models of Operations Research*, 36:517–545, 1992.

[119] P. Marcotte and D. Zhu. Exact and inexact penalty methods for the generalized bilevel programming problem. Technical Report CRT-920, Centre de Recherche sur les Transports, 1992.

[120] R. Mathieu, L. Pittard, and G. Anandalingam. Genetic algorithm based approach to bi-level linear programming. *RAIRO: Recherche Operationelle*, (forthcoming).

[121] M. Mesanovic, D. Macko, and Y. Takahara. *Theory of hierarchical, multilevel systems*. Academic Press, New York and London, 1970.

[122] T. Miller, T. Friesz, and R. Tobin. Heuristic algorithms for delivered price spatially competitive network facility location problems. *Annals of Operations Research*, 34:177–202, 1992.

[123] J. Moore. *Extensions to the multilevel linear programming problem.* PhD thesis, Department of Mechanical Engineering, University of Texas, Austin, 1988.

[124] J. Moore and J. Bard. The mixed integer linear bilevel programming problem. *Operations Research*, 38:911–921, 1990.

[125] J. Morgan. Constrained well-posed two-level optimization problems. Technical Report CRM-1576, Centre de recherches mathematiques, 1988.

[126] J. Morgan and P. Loridan. Approximation of the Stackelberg problem and applications in control theory. In G. Di Pillo, editor, *Control application of nonlinear programming and optimization: Proceedings of the Fifth IFAC Workshop, Capri, Italy 11–14 June*, pages 121–124, 1985.

[127] S. Narula and A. Nwosu. A dynamic programming solution for the hierarchical linear programming problem. Technical Report 37–82, Department of Operations Research and Statistics, Rensselaer Polytechnic Institute, 1982.

[128] S. Narula and A. Nwosu. Two-level hierarchical programming problems. In P. Hansen, editor, *Essays and surveys on multiple criteria decision making*, pages 290–299. Springer-Verlag, Berlin, 1983.

[129] S. Narula and A. Nwosu. An algorithm to solve a two-level resource control preemptive hierarchical programming problem. In P. Serafini, editor, *Mathematics of multiple-objective programming.* Springer-Verlag, Berlin, 1985.

[130] P. Neittaanmäki and A. Stachurski. Solving some optimal control problems using the barrier penalty function method. In H.-J. Sebastian and K. Tammer, editors, *Proceedings of the 14th IFIP Conference on System Modelling and Optimization, Leipzig 1989*, pages 358–367. Springer, 1990.

[131] A. Nwosu. *Pre-emptive hierarchical programming problem: a decentralized decision model.* PhD thesis, Department of Operations Research and Statistics, Rensselaer Polytechnic Institute, 1983.

[132] H. Önal. Computational experience with a mixed solution method for bilevel linear/quadratic programs. 1992. (Preprint from the University of Illinois at Urbana-Champaign).

[133] H. Önal. A modified simplex approach for solving bilevel linear programming problems. *European Journal of Operational Research*, 67:126–135, 1993.

[134] J. Outrata. On the numerical solution of a class of Stackelberg problems. *ZOR - Methods and Models of Operations Research*, 34:255–277, 1990.

[135] J. Outrata. Necessary optimality conditions for Stackelberg problems. *Journal of Optimization Theory and Applications*, 76:305–320, 1993.

[136] J. Outrata. On optimization problems with variational inequality constraints. *SIAM Journal on Optimization*, 4:340–357, 1994.

[137] J. Outrata and J. Zowe. A numerical approach to optimization problems with variational inequality constraints. Technical Report DFG (German Scientific Foundation) Research Report 463, University of Bayreuth, 1993.

[138] G. Papavassilopoulos. Algorithms for static Stackelberg games with linear costs and polyhedral constraints. In *Proceedings of the 21st IEEE Conference on Decisions and Control*, pages 647–652, 1982.

[139] F. Parraga. *Hierarchical programming and applications to economic policy.* PhD thesis, Systems and Industrial Engineering Department, University of Arizona, 1981.

[140] G. Savard. *Contributions à la programmation mathématique à deux niveaux.* PhD thesis, École Polytechnique, Université de Montréal, 1989.

[141] G. Savard and J. Gauvin. The steepest descent direction for the nonlinear bilevel programming problem. *Operations Research Letters*, 15:275–282, 1994.

[142] G. Schenk. A multilevel programming model for determining regional effluent charges. Master's thesis, Department of Industrial Engineering, State University of New York at Buffalo, 1980.

[143] R. Segall. Bi-level geometric programming: a new optimization model. 1989. (Preprint from the Department of Mathematics, University of Lowell Olsen Hall).

[144] J. Shaw. A parametric complementary pivot approach to multilevel programming. Master's thesis, Department of Industrial Engineering, State University of New York at Buffalo, 1980.

[145] H. Sherali. A multiple leader Stackelberg model and analysis. *Operations Research*, 32:390–404, 1984.

[146] K. Shimizu. *Two-level decision problems and their new solution methods by a penalty method*, volume 2 of *Control science and technology for the progress of society*, pages 1303–1308. IFAC, 1982.

[147] K. Shimizu and E. Aiyoshi. A new computational method for Stackelberg and min-max problems by use of a penalty method. *IEEE Transactions on Automatic Control*, 26:460–466, 1981.

[148] K. Shimizu and E. Aiyoshi. Optimality conditions and algorithms for parameter design problems with two-level structure. *IEEE Transactions on Automatic Control*, 30:986–993, 1985.

[149] M. Simaan. Stackelberg optimization of two-level systems. *IEEE Transactions on Systems, Man, and Cybernetics*, 7:554–557, 1977.

[150] H. Stackelberg. *The theory of the market economy.* Oxford University Press, 1952.

[151] S. Suh and T. Kim. Solving nonlinear bilevel programming models of the equilibrium network design problem: a comparative review. *Annals of Operations Research*, 34:203–218, 1992.

[152] C. Suwansirikul, T. Friesz, and R. Tobin. Equilibrium decomposed optimization: a heuristic for the continuous equilibrium network design problem. *Transportation Science*, 21:254–263, 1987.

[153] K. Tammer. Two-level optimization with approximate solutions in the lower level. *ZOR - Mathematical Methods of Operations Research*, 41, 1995.

[154] T. Tanino and T. Ogawa. An algorithm for solving two-level convex optimization problems. *International Journal of Systems Science*, 15:163–174, 1984.

[155] R. Tobin and T. Friesz. Spatial competition facility location models: definition, formulation and solution approach. *Annals of Operations Research*, 6:49–74, 1986.

[156] H. Tuy, A. Migdalas, and P. Värbrand. A global optimization approach for the linear two-level program. *Journal of Global Optimization*, 3:1–23, 1993.

[157] H. Tuy, A. Migdalas, and P. Värbrand. A quasiconcave minimization method for solving linear two-level programs. *Journal of Global Optimization*, 4:243–263, 1994.

[158] G. Ünlü. A linear bilevel programming algorithm based on bicriteria programming. *Computers and Operations Research*, 14:173–179, 1987.

[159] L. Vicente. Bilevel programming. Master's thesis, Department of Mathematics, University of Coimbra, 1992. Written in Portuguese.

[160] L. Vicente and P. Calamai. Geometry and local optimality conditions for bilevel programs with quadratic strictly convex lower levels. Technical Report #198-O-150294, Department of Systems Design Engineering, University of Waterloo, 1994.

[161] L. Vicente, G. Savard, and J. Júdice. The discrete linear bilevel programming problem. To appear in Journal of Optimization Theory and Applications.

[162] L. Vicente, G. Savard, and J. Júdice. Descent approaches for quadratic bilevel programming. *Journal of Optimization Theory and Applications*, 81:379–399, 1994.

[163] U. Wen. *Mathematical methods for multilevel linear programming.* PhD thesis, Department of Industrial Engineering, State University of New York at Buffalo, 1981.

[164] U. Wen. The "Kth-Best" algorithm for multilevel programming. 1981. (Preprint from the Department of Operations Research, State University of New York at Buffalo).

[165] U. Wen. A solution procedure for the resource control problem in two-level hierarchical decision processes. *Journal of Chinese Institute of Engineers*, 6:91–97, 1983.

[166] U. Wen and W. Bialas. The hybrid algorithm for solving the three-level linear programming problem. *Computers and Operations Research*, 13:367–377, 1986.

[167] U. Wen and S. Hsu. A note on a linear bilevel programming algorithm based on bicriteria programming. *Computers and Operations Research*, 16:79–83, 1989.

[168] U. Wen and S. Hsu. Linear bi-level programming problems – a review. *Journal of the Operational Research Society*, 42:125–133, 1991.

[169] U. Wen and S. Hsu. Efficient solutions for the linear bilevel programming problem. *European Journal of Operational Research*, 62:354–362, 1992.

[170] U. Wen and Y. Yang. Algorithms for solving the mixed integer two-level linear programming problem. *Computers and Operations Research*, 17:133–142, 1990.

[171] D. White and G. Anandalingam. A penalty function approach for solving bi-level linear programs. *Journal of Global Optimization*, 3:397–419, 1993.

[172] S. Wu, P. Marcotte, and Y. Chen. A cutting plane method for linear bilevel programs. 1993. (Preprint from the Centre de Recherche sur les Transports).

[173] J. Ye. Necessary conditions for bilevel dynamic optimization problems. *SIAM Journal on Control and Optimization*, 33:1208–1223, 1995.

[174] J. Ye and D. Zhu. Optimality conditions for bilevel programming problems. Technical Report DMS-618-IR, Department of Mathematics and Statistics, University of Victoria, 1993. 1992, Revised 1993.

[175] J. Ye, D. Zhu, and Q. Zhu. Generalized bilevel programming problems. Technical Report DMS-646-IR, Department of Mathematics and Statistics, University of Victoria, 1993.