Multi-material 3D printing using stereolithography: an optimization approach

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Abstract. Composite multi-material components are increasingly sought in several markets. Such components allow for the shrinking of systems, operation and build simplification, with improved reliability of the associated functions. The current work addresses the construction of hybrid parts through the use of an additive process that adds polymer resin to another, pre-existing object (metallic, polymeric, ceramics, or others). One of the processes that is most commonly used with three-dimensional printing is the stereolithography (SL). In this case the printing is done layer by layer, and a laser beam or a section projected by a DLP system (Digital Light Processing) is applied in multiple directions, in order to polymerizing the selected areas of the liquid polymer resin like a two-dimensional printer would. This paper is focused on the laser beam approach. The DLP approach is under development and will be described in next works. When applying a second material, though, the pre-existing structure may prevent the laser beam or the DLP light from reaching the printing layer, thus producing "shaded" regions. The current work describes an evolution of the SL process that can use more than one laser emitter or DLP system, in order to overcome the shaded regions. The problem is handled in two main parts, first the projection of the laser or DLP on the printing layer is analyzed, second the determination of the minimum number of required laser or DLP emitters is formulated as a set covering problem. Finally, the results of empirical tests of the proposed approach for a case study are reported and discussed.

Introduction

Three-dimensional printing, or 3D printing, is an additive process for rapid free form manufacturing, where the final object is created by the addition of successive thin layers of material. Each layer corresponds to a cross-section of the object to be constructed, and the printer draws each layer as if it were a two-dimensional print. More details on these processes can be found in [1, 3]. The technology and materials required for this type of process started being developed at the end of the last century, in the 80's. Several improvements have been made to 3D printing methods ever since their origin, and nowadays printers can be purchased by reasonable prices.

One of the technologies for 3D printing is SL [4]. In this case a liquid polymer is added layer by layer, and each layer is exposed to ultraviolet, infrared, visible or other laser and projected structured light. Only the zone of the polymer the laser beam or projection reaches is polymerized. The platform that supports the model moves to get ready for printing the next layer. Using a process analogous to SL, we intend to print an object in which the polymer covers a previously constructed 3D metal, ceramic or other grid structure. The use of a second material in the printing, for example the existence of an a priori constructed metal grid that serves as support for the polymer, brings additional difficulties. Figure 1 describes the full NEXT.parts process. The steps identified with (3c) and then (8) are those described in this work. All laser trajectories or light projections are calculated on the NEXT.SL software module and are implemented on the NEXT.SL additive manufacturing system. As mentioned before, the system could be implemented by using one or several laser beams or projections from DLPs. However, this work is focused on the laser version only.

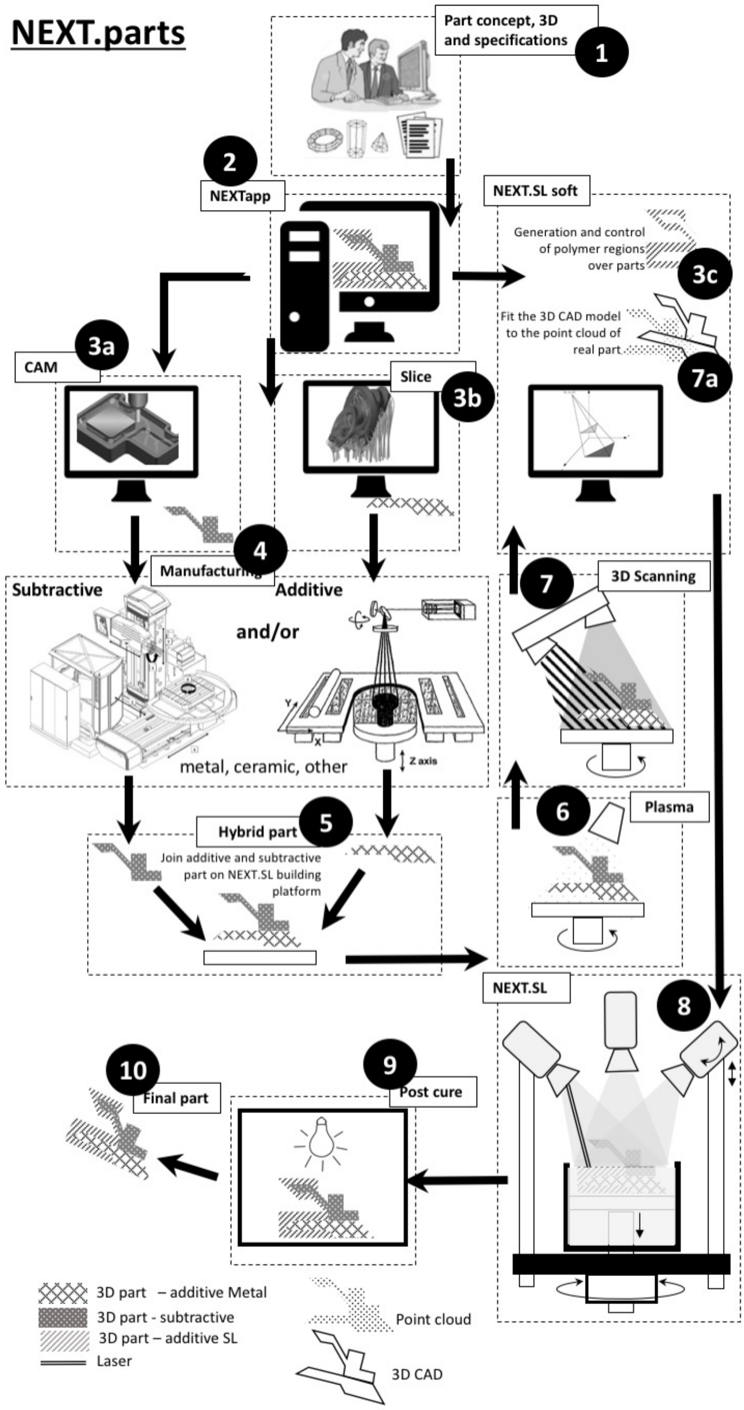


Figure 1: NEXT.parts system

Traditional processes could use a single fixed laser or DLPs which guides its beam in the desired directions of the printing platform. If a metal structure is pre-installed, it may block the light from the laser, thus creating shaded areas that cannot be polymerized. The current work is dedicated to studying the possibility of overcoming this issue by placing additional galvanometer mirror scanners along the walls of the printer. These galvanometer mirror scanners are assumed to be in positions that depend on the object. These positions are fixed from the beginning of the printing process, however the laser beam reflected by each galvanometer scanner can be oriented with the goal of reaching the shaded areas.

The contribution of this paper is bifold and aims to answer the following questions:

- to identify the shaded areas at the printing level, given a pre-existing metal structure and a position for the galvanometer mirror scanner, and

- on a second step, to find the least number of those scanners that ensure the full printing of a given object for a given pre-existing metal structure, and to know where to install them.

The first question is addressed using geometric arguments to calculate how a point in the metal structure is projected on the printing platform from a certain origin of the laser beam. This information, combined with the knowledge of the obstacles the laser light may have, is used to construct a printing space coverage matrix. Similar arguments are used to generalize this projection for projecting the circle formed at the origin of the laser beam over the printing layer. This result is used later to estimating the distortion of the laser beam when polymerizing the polymer resin. The distortion of the laser is related with the angle of the beam with respect to the printing platform and is used to assess the quality of the solution. To answer the second question it is shown that the problem can be formulated as a set covering problem, which is a classical linear integer (binary) optimization problem, using the set covering matrix that results from the previous step. The proposed approach is tested for a simple case study.

The remainder of the paper is organized as follows. In the next section, we introduce the general setting of the printing process, calculate the projection of a point on a horizontal plane taking possible obstacles into account, and explain how a circle of laser light is projected under the same conditions. Afterwards, we describe how to obtain the coverage matrix, which represents the positions of the object to print that are reachable from a laser light emitted from a certain position. Then a linear binary program is formulated to find the best location for the laser emitters, based on the latter matrix. Finally, computational experiments are presented, while conclusions and future lines of work are drawn in the last section.

Laser beam projection

In the following it is assumed that a metal structure, with a known model, is installed in the printer from the beginning of the printing process. The purpose is then to coat that structure with polymer resin. The metal structure is placed on a movable platform at the base of the printer, where the polymer is polymerized. The platform is moved down after each additional layer of polymer is finished. Moreover, we consider that there always is a device placed at the center of the top of the printer, which is able to emit a laser light in any direction. Additional devices that emit or reflect a laser light, like galvanometer mirror scanners, may be installed on the side walls of the printer in order to make possible the complete construction of the object. These devices will be called emitters. For the sake of the stability of the system, the emitters are considered to be fixed along the printing process, while they may reflect the laser beam at specific angles.

As already explained, the object is divided into several layers that are successively added all along the printing process. The two-dimensional space where the layer is constructed is partitioned into uniform $l \times l$ squares, called voxels, in order to specify which ones should be reached by laser light. The 3D coordinate system is considered so that the printing platform, where the new layer is added, coincides with the $x0y$ plane and the coordinates of the centers of the voxels of the layer to be added are given by $(r, s, 0)$, for $r = 1, \dots, l$, $s = 1, \dots, l$. The third coordinate of the system concerns the height with respect to the printing platform, and it is necessary as a reference for the coordinates of the emitters and the points used to define the metal grid.

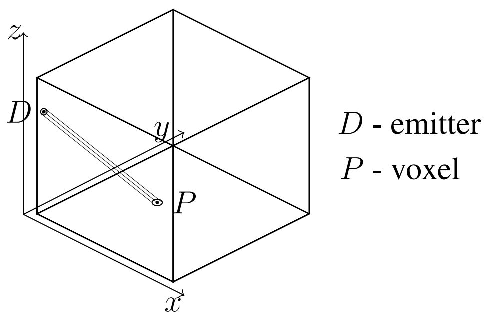


Figure 2: Printing area

The metal grid is assumed to be formed by segments, each one defined by the coordinates of its extreme points. For the sake of simplicity, it is assumed that these are straight line segments. It should be stressed that what happens at the $x0y$ plane level, and under, is not relevant, neither are any points at the level of the emitter, or above. Let $D = (d_1, d_2, d_3)$ be the position of an emitter and $P = (x, y, z)$ be the position of an object voxel, as depicted in Figure 2. Thus, to determine the voxels $(r, s, 0)$ reached by D, it is enough to analyze the points P of the line segments forming the grid such that $0 < z < d_3$. We assume that there are q line segments in these conditions and denote their extreme points by $U(i)$ and $V(i)$, for $i = 1, \ldots, q$. We consider that the segment formed by the projection of the two extreme points in the layer is given by a discrete set of points.

Considering that $D = (d_1, d_2, d_3)$ is the position of the emitter and $U = (u_1, u_2, u_3)$ is the position of an extreme point of a line segment, we want to know the projection of point U on the printing platform. Let $(Proj_x(U), Proj_y(U))$ denote the new point. The vector that goes from point D to point U is $\overrightarrow{(u_1 - d_1, u_2 - d_2, u_3 - d_3)}$. Multiplying this vector by $\frac{d_3}{d_3 - u_3}$, we obtain the vector

$$\overrightarrow{\left(\frac{d_3}{d_3 - u_3}(u_1 - d_1), \frac{d_3}{d_3 - u_3}(u_2 - d_2), -d_3\right)},$$

therefore, by adding this vector to point D, we obtain

$$Proj_x(U) = d_1 + \frac{d_3}{d_3 - u_3}(u_1 - d_1) \quad \text{and} \quad Proj_y(U) = d_2 + \frac{d_3}{d_3 - u_3}(u_2 - d_2). \tag{1}$$

We now analyze the distortion of the laser light when it reaches the printing layer. In particular, the area reached by the laser beyond the voxel and the area of the voxel that remains to be reached are calculated. This areas will be used to evaluate the quality of a solution.

Two angles define in which direction an emitter reaches a voxel, α e θ. The angle α is measured in the $x0y$ plane, and it is formed between the semi-straight line parallel to the x axis with positive direction and the semi-straight line starting at D and going through P. The angle θ corresponds to the angle formed between the beam emitted downward and the horizontal plane that passes through D. Both are depicted in Figure 3.

At the emitter the laser beam used for printing has the shape of a circle, however it reaches the printing surface as an ellipse. In what follows, we study the shape of this ellipse and its position with respect to the voxels, considering that r is the radius of the circle of laser beam.

If $d_1 = p_1$ and $d_2 = p_2$, the light is emitted vertically and the ellipse degenerates into a circle of radius r. This case is trivial and is excluded from the following analysis.

The length of the smaller semi-axis of the ellipse, denoted by b, is always $b = r$. On the other hand, the larger semi-axis, denoted by a, depends on the positions of D and P, and is oriented according to the angle α – Figure 4. Based on the coordinates of D and P, and on Figure 3, assuming that $d_1 < p_1$

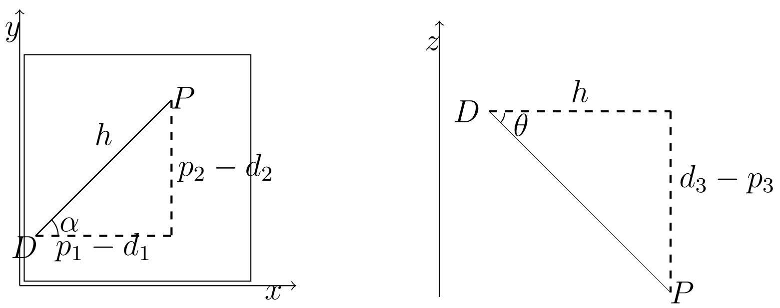


Figure 3: Angles α and θ

and $d_2 \leq p_2$,

$$\tan(\alpha) = \frac{p_2 - d_2}{p_1 - d_1} \Leftrightarrow \alpha = \arctan\left(\frac{p_2 - d_2}{p_1 - d_1}\right).$$

Considering that $\arctan$ assumes values in $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$, this formula can be generalized as

$$\alpha = \begin{cases} \arctan(\frac{p_2 - d_2}{p_1 - d_1}) & \text{if } d_1 < p_1 \text{ and } d_2 \leq p_2 \\ 2\pi + \arctan(\frac{p_2 - d_2}{p_1 - d_1}) & \text{if } d_1 < p_1 \text{ and } d_2 > p_2 \\ \pi + \arctan(\frac{p_2 - d_2}{p_1 - d_1}) & \text{if } d_1 > p_1 \\ \frac{\pi}{2} & \text{if } d_1 = p_1 \text{ and } d_2 < p_2 \\ \frac{3\pi}{2} & \text{if } d_1 = p_1 \text{ and } d_2 > p_2 \end{cases}$$

Now, to determine a it is necessary to calculate the distance h in Figure 3, and by the Pythagorean Theorem it follows that

$$h = \sqrt{(p_1 - d_1)^2 + (p_2 - d_2)^2}.$$

Based on the Figures 3 and 4, we have

$$\tan(\theta) = \frac{d_3 - p_3}{h} \Leftrightarrow \theta = \arctan\left(\frac{d_3 - p_3}{h}\right),$$

and, thus, $a = \frac{r}{\sin(\theta)}$.

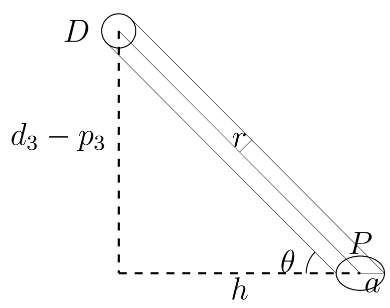


Figure 4: Distortion of the laser light when reaching a voxel

It is assumed that the side of each voxel has length 1, so its area is 1 as well. After knowing which emitter reaches the voxel, the angles α and θ can be calculated. We consider that the center of the laser is pointed toward the center of the voxel, therefore, a zone beyond the target voxel can be reached and part of the target voxel may be unreached by this laser light. The areas of these zones will be called outer area, A_{out}, and inner area, A_{in}, respectively – Figure 5.

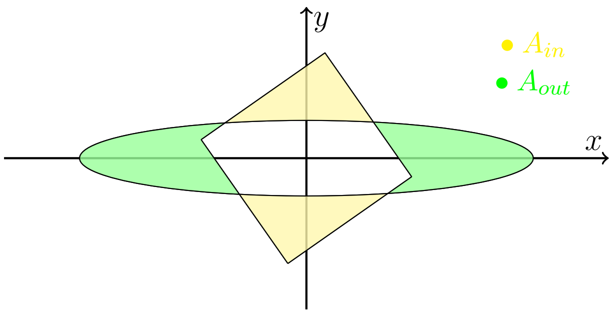


Figure 5: Areas to calculate

It is assumed that the laser beam used is completely contained in a voxel, that is, $2r \leq 1$. Otherwise, the laser may not be accurate enough for printing. As seen earlier, the ellipse is such that

$$a = \frac{r}{\sin(\theta)} \quad \text{and} \quad b = r.$$

The angle $\alpha \in]0, 2\pi[$ determines the orientation of the ellipse with respect to the voxel, but for our study it is enough to know the angle γ given by the rotation of the voxel when its edges are parallel to the axes of the ellipse. This angle can be measured in the direct or the reverse direction, thus we consider both cases and $\gamma \in]0, \frac{\pi}{4}[$. The two cases are symmetrical with respect to one of the axes of the ellipse, so γ can be obtained from α as

$$\gamma = \begin{cases} \alpha & \text{if } 0 \leq \alpha \leq \frac{\pi}{4} \\ \frac{\pi}{2} - \alpha & \text{if } \frac{\pi}{4} < \alpha \leq \frac{\pi}{2} \\ \alpha - \frac{\pi}{2} & \text{if } \frac{\pi}{2} < \alpha \leq \frac{3\pi}{4} \\ \pi - \alpha & \text{if } \frac{3\pi}{4} < \alpha \leq \pi \\ \alpha - \pi & \text{if } \pi < \alpha \leq \frac{5\pi}{4} \\ \frac{3\pi}{2} - \alpha & \text{if } \frac{5\pi}{4} < \alpha \leq \frac{3\pi}{2} \\ \alpha - \frac{3\pi}{2} & \text{if } \frac{3\pi}{2} < \alpha \leq \frac{7\pi}{4} \\ 2\pi - \alpha & \text{if } \frac{7\pi}{4} < \alpha < 2\pi \end{cases}$$

For convenience, and without loss of generality, in the following it is considered that the coordinate system is aligned with the axes of the ellipse, as represented in Figure 6. Depending on the position of the emitter, the sides of the voxel to reach may be parallel to the axes, when $\gamma = 0^{\circ}$, or suffer a rotation equal to the angle γ.

Because of the symmetry shown in Figure 5, the outer region can be split into two equal regions. To calculate the inner area, the area of the inner ellipse is determined and this value is subtracted from the voxel area. Therefore, to determine A_{out} and A_{in} it is enough to calculate one of the lateral areas outside the voxel. Without loss of generality, the area on the right is calculated.

It is important to note that the ellipse may be fully contained in voxel. In this case $A_{out} = 0$ and only A_{in} is determined. For the calculation of the desired areas and to check if the ellipse is circumscribed in the voxel, it is necessary to check whether the ellipse intersects the green or the red edges in Figure 6. When they exist, the intersection points will be denoted by $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$.

The reduced equation of the ellipse is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2}$$

and the reduced equation of the line containing the green edge, for $\gamma > 0$, is given by

$$y = -\frac{1}{\tan(\gamma)}x + \frac{0.5}{\sin(\gamma)}. \tag{3}$$

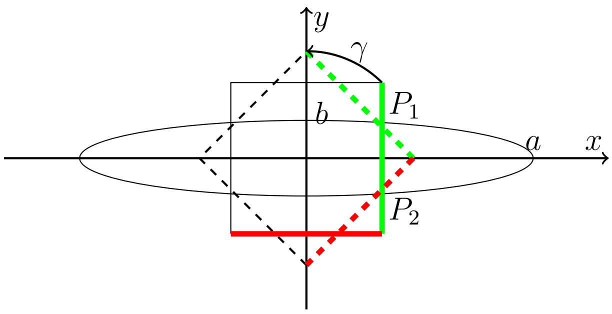


Figure 6: Projection of the laser light in a voxel

The reduced equation of the line containing the red edge, perpendicular to the previous one, is given by

$$y = \tan(\gamma)x - \frac{0.5}{\cos(\gamma)}. \tag{4}$$

If $\gamma = 0$, the ellipse can only intersect the green edge, and it is easy to conclude that this only happens if the length of the major axis of the ellipse exceeds the length of the side of the voxel, that is, if $2a \geq 1$. In this case, we have $x = 0.5$ and by equation (2),

$$P_1 = (0.5, \frac{b}{a}\sqrt{a^2 - 0.5^2}) \text{ and } P_2 = (0.5, -\frac{b}{a}\sqrt{a^2 - 0.5^2}).$$

When $\gamma > 0$, if the voxel and the ellipse intersect, at least one of the intersections will be with the green edge. This case is checked next. Combining equations (2) and (3), it can be concluded that this only happens if

$$\frac{1}{\tan^2(\gamma)\sin^2(\gamma)} - 4\Big(\frac{1}{\tan^2(\gamma)} + \frac{a^2}{b^2}\Big)\Big(\frac{0.25}{\sin^2(\gamma)} - b^2\Big) > 0.$$

If this condition is not satisfied, the ellipse is completely inscribed in the voxel. Otherwise, the point $P_1 = (x_1, y_1)$ corresponds to the solution of the intersection of the equations with the smallest abscissa, therefore

$$x_1 = \frac{\frac{1}{\tan(\gamma)\sin(\gamma)} - \sqrt{\frac{1}{\tan^2(\gamma)\sin^2(\gamma)} - 4\Big(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\Big)\Big(\frac{0.25}{\sin^2(\gamma)} - b^2\Big)}}{2\Big(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\Big)}$$

and

$$y_1 = -\frac{1}{\tan(\gamma)}x_1 + \frac{0.5}{\sin(\gamma)}.$$

If the ellipse intersects the voxel it is necessary to determine point P_2 as well. Two cases are considered:

- either the ellipse intersects the green edge,
- or the ellipse intersects the red edge,

One way to determine which of the two edges it intersects is to check whether the greatest abscissa of the intersection points of the curves (2) and (3) is bigger than the abscissa of the vertex formed by the green and the red edges, $V = (x_3, y_3)$. Combining the equations (3) and (4) leads to

$$x_3 = \frac{1 + \tan(\gamma)}{2\cos(\gamma)(\tan^2(\gamma) + 1)}. \tag{5}$$

Therefore, if

$$\frac{\frac{1}{\tan(\gamma)\sin(\gamma)} - \sqrt{\frac{1}{\tan^2(\gamma)\sin^2(\gamma)} - 4\left(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\right)\left(\frac{0.25}{\sin^2(\gamma)} - b^2\right)}}{2\left(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\right)} \leq \frac{1+\tan(\gamma)}{2\cos(\gamma)(\tan^2(\gamma)+1)},$$

then P_2 is the point where the ellipse and the green edge meet. Otherwise, P_2 is common to the ellipse and the edge in red. In the first case we have that $P_2 = (x_2, y_2)$ is such that

$$x_2 = \frac{\frac{1}{\tan(\gamma)\sin(\gamma)} + \sqrt{\frac{1}{\tan^2(\gamma)\sin^2(\gamma)} - 4\left(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\right)\left(\frac{0.25}{\sin^2(\gamma)} - b^2\right)}}{2\left(\frac{1}{\tan^2(\gamma)} + \frac{b^2}{a^2}\right)}$$

and

$$y_2 = -\frac{1}{\tan(\gamma)}x_2 + \frac{0.5}{\sin(\gamma)}.$$

In the second case, it holds that

$$x_2 = \frac{\frac{\tan(\gamma)}{\cos(\gamma)} - \sqrt{\frac{\tan^2(\gamma)}{\cos^2(\gamma)} - 4\left(\tan^2(\gamma) + \frac{b^2}{a^2}\right)\left(\frac{0.25}{\cos^2(\gamma)} - b^2\right)}}{2\left(\tan^2(\gamma) + \frac{b^2}{a^2}\right)}$$

and

$$y_2 = \tan(\gamma)x_2 - \frac{0.5}{\cos(\gamma)}.$$

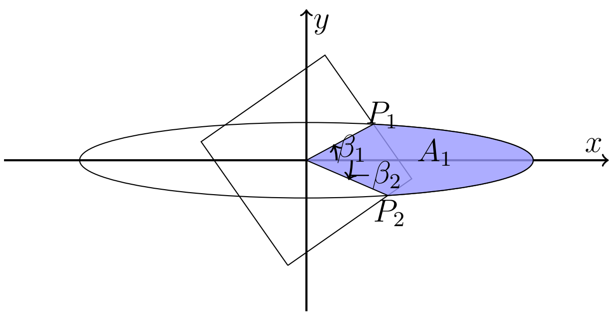


Figure 7: Area of ellipse's section

After the analysis of the intersections, and knowing that they exist, that is, that the ellipse goes beyond the square, we start by calculating the area of the section of the ellipse shown in blue in Figure 7, A_1. The angles β_1 and β_2 in the plot are obtained from the coordinates of P_1 and P_2 as

$$\beta_1 = \arctan\left(\frac{y_1}{x_1}\right) \text{ and } \beta_2 = \arctan\left(\frac{y_2}{x_2}\right).$$

Then,

$$A_1 = \frac{ab}{2}\left(\arctan\left(\frac{a}{b}\tan(\beta_1)\right) - \arctan\left(\frac{a}{b}\tan(\beta_2)\right)\right).$$

The next step is to subtract A_2, corresponding to the part inside the voxel, from A_1. Two situations are analyzed for calculating A_2: either the ellipse intersects only one edge of the voxel – Figure 8a, or the ellipse intersects two distinct edges – Figure 8b. In both figures the area to be calculated is represented in red. In the first case

$$A_2 = \frac{|x_2y_1 - x_1y_2|}{2},$$

whereas in the second

$$A_2 = \frac{|x_3y_1 - x_1y_3|}{2} + \frac{|x_2y_3 - x_3y_2|}{2}.$$

Replacing (5) in the equation of one of the edges, for instance the edge with equation (4), results in

$$y_3 = \tan(\gamma)x_3 - \frac{0.5}{\cos(\gamma)}.$$

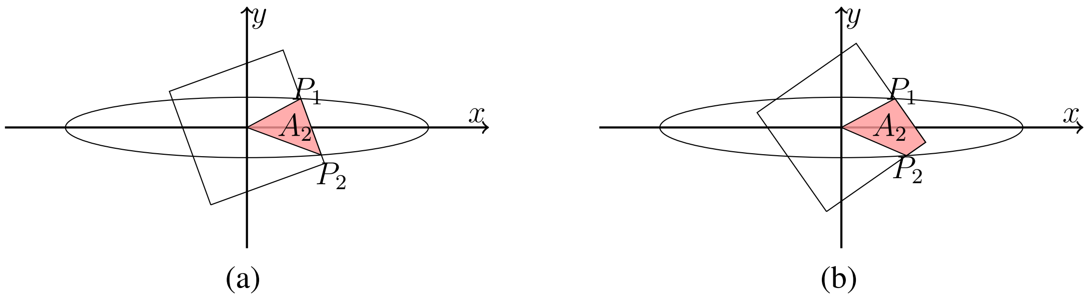


Figure 8: Area of section of ellipse within voxel

After all cases have been analyzed, and recalling that the area of an ellipse is given by $A = \pi ab$, we conclude that:

1. if the ellipse is circumscribed in the voxel, then

$$A_{out} = 0 \quad \text{and} \quad A_{in} = 1 - A,$$

2. if the ellipse is not limited to the voxel, then

$$A_{out} = 2(A_1 - A_2) \quad \text{and} \quad A_{in} = 1 - (A - A_{out}) = 1 - \pi ab + 2(A_1 - A_2).$$

Finally, it should be noted that the outer areas are only relevant for the distortion of an object for voxels on its border.

Multi-material three-dimensional printing problem

In the following we describe an algorithm to determine the voxels that are reachable from an emitter installed at a given position. It is assumed that a set of m voxels have to be reached by the laser light, which may be sent from a set of n possible emitter positions, or simply n emitters. One of the emitters is placed at the central position of the top wall of the printer. The remaining are $n - 1$ emitters the positions of which have to be determined.

With the purpose of knowing the set of voxels that can be reached from each position, we define the emitters' coverage matrix, $A = [a_{ij}]_{i=1,\dots,m;j=1,\dots,n}$, where

$$a_{ij} = \begin{cases} 1 & \text{if the emitter } j \text{ can reach the voxel } i \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, \dots, m$, $j = 1, \dots, n$.

Let us assume that the printing is done in p layers and that there are m_k voxels to reach at each of them, $k = 1, \ldots, p$. Obviously, $m_1 + m_2 + \ldots + m_p = m$. Consider a block partition of matrix A, such that each submatrix $A_k \in \{0, 1\}^{m_k \times n}$ contains the lines of matrix A corresponding to the voxels at layer k, $k = 1, \ldots, p$. The column j of matrix A_k corresponds to the voxels of layer k reachable by emitter j, $j = 1, \ldots, n$. Algorithm 1 outlines the steps needed to determine the column j of matrix A_k. In the pseudo-code, B denotes a matrix that is an auxiliary variable. The positions of matrix B are initialized to 1 (lines 1-2). Then those that correspond to shaded zones are updated to 0 (line 6).

Algorithm 1: Calculation of column j of matrix A_k

1 **for** $r = 1, \ldots, l$ **do**
2   **for** $s = 1, \ldots, l$ **do** $b_{rs} \leftarrow 1$
3 **for** $i = 1, \ldots, q$ **do**
4     $(x_1, y_1) \leftarrow (Proj_x(U(i)), Proj_y(U(i))$
5     $(x_2, y_2) \leftarrow (Proj_x(V(i)), Proj_y(V(i))$
6   **for** $(r, s) \in \overline{(x_1, y_1)(x_2, y_2)}$ **do** $b_{rs} \leftarrow 0$
7 **for** $i = 1, \ldots, m_k$ **do**
8     $(r, s) \leftarrow$ coordinates of the voxel center i to be reached by the laser light
9     $a_{ij} \leftarrow b_{rs}$

Once the coverage matrix of emitters, A, is known, we need to know where to install the new emitters so that the full object can be printed and the number of emitters is the least possible. This problem can be formulated as a set covering problem, as follows.

Let x_j be binary decision variables such that

$$x_j = \begin{cases} 1 & \text{if the emitter in position } j \text{ is installed} \\ 0 & \text{otherwise} \end{cases}$$

for $j = 1, \ldots, n$. Because there is always an emitter on the top of the printer, $x_1 = 1$ and the total number of emitters to be used, which is to be minimized, is given by

$$\sum_{j=2}^{n} x_j.$$

In addition, a solution is feasible if any voxel $i = 1, \ldots, m$ can be reached by at least one emitter, that is, if

$$\sum_{j=1}^{n} a_{ij} x_j \geq 1, \quad i = 1, \ldots, m.$$

Then, the linear binary problem to solve is

$$\begin{array}{ll} \min & \sum\limits_{j=2}^{n} x_j \\ \text{subject to} & Ax \geq 1 \\ & x_1 = 1 \\ & x \in \{0, 1\}^n \end{array} \tag{6}$$

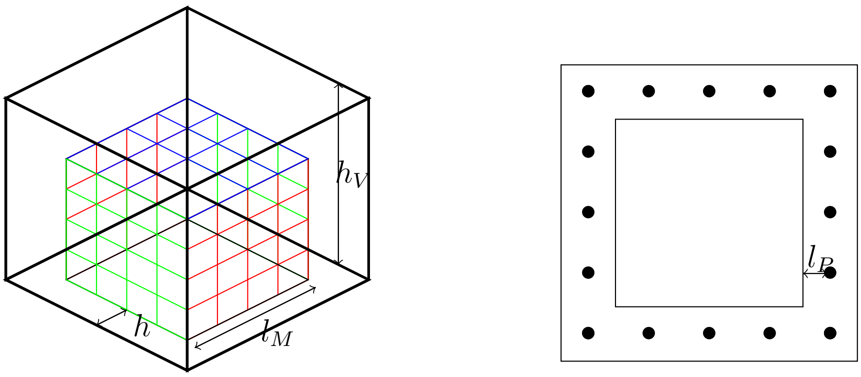
The optimal value of problem (6) is the number of emitters required for the side walls of the printer to complete the printing. Its optimal solution provides the positions where these emitters should be installed, given that these correspond to the indices j such that $x_j = 1$. As mentioned before this is a set covering problem where the lines of matrix A, the object voxels, have to be covered by at least a column of A, that is, an emitter. The set covering problem has been shown to be NP-complete [5]. Nevertheless, its broad application in many areas has justified the attention of many researchers. The interest reader may consult a survey on this topic by Caprara, Toth and Fischetti [2].

Once the position of the emitters is known, and fixed when the object starts being printed, the choice of the emitter to reach a particular voxel is done by maximizing θ, the incidence angle of the laser light. As seen earlier, this angle affects how the laser light reaches the printing layer and the distortion of the final object.

Computational experiments

In this section the proposed methodology to solve the multi-material 3D printing problem is tested for a case study. The case study consists of constructing a cube without one of the faces, by coating each of the five faces of a uniform metal grid with a polymer resin.

The cube to be constructed, without the base, is depicted in Figure 9a. The grid in the image represents the existing metal grid previously produced by SL and which must be coated with polymer. The top of the cube is shown in blue, the front and the rear faces are shown in red, and the left and the right faces are represented in green.



(a) (b)

Figure 9: (a) Printing area and object to be printed; (b) Intermediate cross section of the object to print

The metal grid is considered to be as thick as the polymer layers, that is, equal to 1. This is also the width of the voxels. Figure 9b represents an intermediate layer of the object to be produced. The variable parameters for printing the cube are:

- The length of each segment of the metal grid, l_M.
- The thickness of the polymer added on each side of the metal grid, l_P.
- The number of divisions of the metal grid, assumed to be uniform, n_M.
- The space between the cube to print and the side walls of the printer, where the emitters can be installed, h.
- The height of the printing area, h_V.

Note that for the top face, the length of the edge of the final cube to be printed, denoted by n_V, corresponds to the addition that is made of polymer, with length l_P, on each side to the length of the edges of the cube of metal given by l_M, that is, we have $n_V = 2l_P + l_M$. For the lateral faces, the length of the edge of the final cube will be given by $l_P + l_M$, since the cube has no lower face, that is, in that zone no polymer is added. In this way, the object is formed by $l_P + l_M$ layers. For the tests presented the two upper layers are excluded. When printing, the top face of the grid is already at the level of the layer being printed or below it and all other metal structure below the printing plane. Therefore, the laser installed on the top of the printer can be used directly. It is considered that the cube is centered on the printing platform.

In what follows the used length unit corresponds to 0.2 millimeters, the length of the side of the voxels. In addition, the printer dimensions were fixed to $h_V = 1250$. The value h varies, since it depends on the size of the object to print. However, h is always chosen so that the printing platform also has size of 1250×1250 units. Moreover, regarding the cube to be printed:

- $n_M = 5, 10, 20$,
- $n_V = 200, 300, 500$,
- $l_P = 1$.

The number of voxels at each layer is given by $n_V \times n_V$. The characteristics of the test problems are summarized in Table 1. All tests were solved using the software packages MATLAB and CPLEX.

Table 1: Test parameters

Test	n_V	n_M	h
T1	200	5	525
T2	200	10	525
T3	200	20	525
T4	300	5	475
T5	300	10	475
T6	300	20	475
T7	500	5	375
T8	500	10	375
T9	500	20	375

In addition to the direct use of the laser, 60 possible locations for additional emitters, distributed along the side walls of the printer, were considered, resulting in a total of 61 possible emitters' position. A minimum height of 500 was imposed when determining the position of the emitters, given that the distortion of the laser beam on the printing layer from lower positions was too high. The obtained results are presented in Table 2, where z represents the optimal value of problem (6), that is, the minimum number of emitters besides the one at the top of the printer. According to this table, between 2 and 6 additional emitters are required for completing the printing. Also, as expected, in general, the larger the number of divisions of the metal grid, the larger is the number of required emitters.

Table 2: Test results

Test	z	Emitters' position
T1	3	(1250, 1250, 1000), (1, 1, 750), (1,1,500)
T2	3	(1, 1, 500), (1250, 1000,500), (1000, 1250, 500)
T3	6	(1, 251, 1000), (1250, 1000, 1000), (1, 1000, 750), (1000, 1, 750), (1, 1, 500), (1250, 1250, 250)
T4	2	(1, 1, 750), (1250, 1250, 500)
T5	3	(1, 501, 1000), (750, 1, 1000), (1250, 1250, 750)
T6	4	(251, 1250, 750), (1, 501, 500), (1250, 750, 500), (750, 1, 500)
T7	2	(1, 1, 750), (1250, 1250, 500)
T8	3	(1, 750, 500), (251, 1, 500), (1000, 1250, 500)
T9	3	(1250, 251, 1000), (251, 1250, 1000), (1,1,500), (1250, 1250, 500)

The determination of the minimum number of emitters to display around the printer was followed by the selection of which one to use to reach each voxel. In this step it was considered a laser beam with a radius of 0.05 millimeters, that is, of 0.25 units. For the same set of tests, the angles α and θ

used by the emitter when reaching each voxel, were calculated. The values of A_{in} and A_{out} for all voxels were also calculated.

Figure 10 shows the minimum, the mean and the maximum values of θ, as well as the minimum, the mean and the maximum values of the percentage relative A_{in}, as a measure of the printing quality. The values A_{out} were always 0, the area outside the voxels was never reached. The mean values of A_{in} were greater than 69% for all tests. This can be explained by the assumption that the laser beam reaches only the center of the voxels and because the voxels' side has twice the diameter of the laser beam. On the one hand, working with smaller voxels would result in a reduction of this area, but on the other it would also increase the problem's dimension. The fact that A_{out} was 0 for all tests can also be due to the restriction that emitters are at 500 units minimum height.

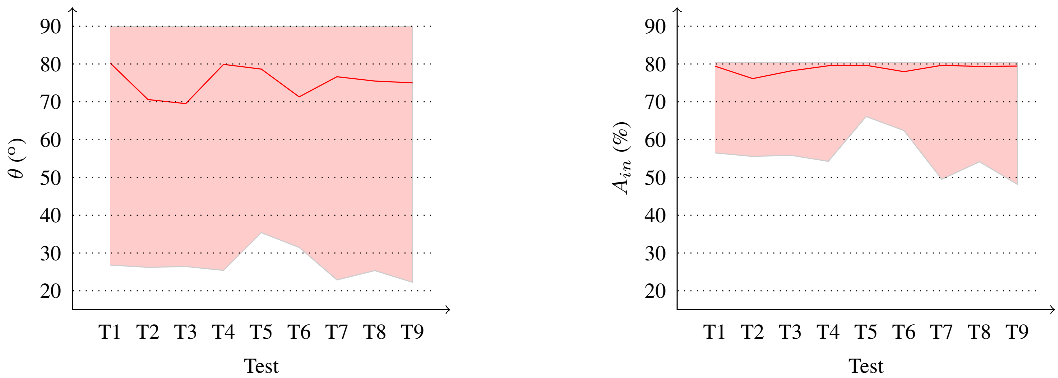


Figure 10: Minimum, mean and maximum values for θ and A_{in}

Conclusions

In this work the possibility of solving the multi-material three-dimensional printing problem was studied by placing galvanometer mirror scanners on the side walls of a printer. We started by analyzing which voxels are reached by each emitter and studying how the position of an emitter affects the laser projection on the printing layer. An integer optimization model was proposed for finding the minimum number of laser emitters, as well as their positions on the printer. Finally, a case study was described and the corresponding computational results were presented.

The computational experiments carried out have shown that the use of additional galvanometer scanners may be an alternative for implementing multi-material 3D printing. For all tests, theoretical solutions were obtained that satisfy the imposed constraints. Two parameters were calculated as measures of the quality of the determined solutions, the outer area, formed by the regions polymerized beyond the object voxels, and the inner area, formed by the regions of the object voxels that remained unpolymerized. Both depend on the incidence angle of the laser beam at the printing layers. On the one hand, for the considered case study, the outer area was always null. On the other, the values of the inner area were around 70%. In principle these values could be improved by skipping the simplification that it is enough that the laser reaches the center of the voxels. Also, in practice finishing techniques could be used to overcome high values for A_{out} or small values for A_{in}.

Future work includes performing further experiments for more complex case studies, as well as to studying the possibility of using other printing methodologies different from galvanometer mirror scanners.

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References

[1] M. Burns. *Automated fabrication: improving productivity in manufacturing*. Prentice-Hall, Inc., 1993.

[2] A. Caprara, P. Toth, and M. Fischetti. Algorithms for the set covering problem. *Annals of Operations Research*, 98:353–371, 2000.

[3] I. Gibson, D. Rosen, and B. Stucker. *Additive manufacturing technologies*. Springer, New York, 2015.

[4] P. F. Jacobs. *Rapid prototyping & manufacturing: fundamentals of stereolithography*. Society of Manufacturing Engineers, 1992.

[5] R. M. Karp. Reducibility among combinatorial problems. In *Complexity of computer computations*, pages 85–103. Springer, 1972.