Teoria de Jogos | Frequência/Exame (época normal)
Universidade de Coimbra | 3 páginas
Professor João Soares | 24 de Junho de 2010

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1. Consider the zero-sum two-player game with matrix A below. Past experience in playing the game with Player II enables Player I to arrive at a set of probabilities reflecting his belief of how Player II will play. I thinks that with probabilities 2/5, 2/5, 1/10, and 1/10, II will choose columns 1, 2, 3, and 4, respectively, of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & -0.5 \end{bmatrix}$$

a. Find for I a Bayes strategy p against $(2/5, 2/5, 1/10, 1/10)$ - that is, a best response.

b. Suppose II guesses correctly that I is going to use p. Instruct II on the strategy she should use - that is, find II's Bayes strategy against I's Bayes strategy against $(2/5, 2/5, 1/10, 1/10)$. What, then, will be the outcome of the game?

c. Put yourself on Player I's shoes. You are more likely to believe that II will use all columns except the fourth one in her optimal mixed strategy. Show that this guess will give you the value of the game. What are the minmax strategies for both players?

2. Player II chooses one of the numbers in the set $\{1, 2, 3\}$. Then one of the numbers not chosen is selected at random and shown to Player I. Then Player I guesses which number Player II chose, winning that number if he is correct and winning nothing otherwise.

a. Draw the Kuhn Tree.

b. Describe shortly what are Player I's behavioral strategies for this game.

3. Consider the non-cooperative bimatrix game:

$$(A, B) \equiv \begin{bmatrix} (3,4) & (2,3) & (3,2) \\ (6,1) & (0,2) & (3,3) \\ (4,6) & (3,4) & (4,5) \end{bmatrix}$$

a. Find the safety levels, and the maxmin strategies for both players.

b. Find as many strategic equilibria as you can.

4. Find the NTU-solution and the equilibrium exchange rate for the following fixed threat point game,

$$\begin{bmatrix} (6,3) & (0,0) & (0,0) \\ (1,8) & (4,6) & (0,0) \\ (0,0) & (0,0) & (0,0) \end{bmatrix}$$

5. Consider the weighted majority game with two large parties with 1/3 of the votes each and three smaller parties with 1/9 of the votes each. Find the Shapley-Shubik index value of each party. Is the combined power of the two larger parties greater or less than its proportionate size?

6. Three farms are connected to each other and to the main highway by a series of rough trails as shown in the figure. Each farmer would benefit by having a paved road connecting his farm to the highway. The amounts of these benefits are indicated in square brackets [...]. The costs of paving each section of the trails are also indicated on the diagram. It is clear that no single farmer would find it profitable to build his own road, but a cooperative project would obviously be worthwhile.

a. Determine the characteristic function.

b. Find the Shapley value.

c. Find the nucleolus.

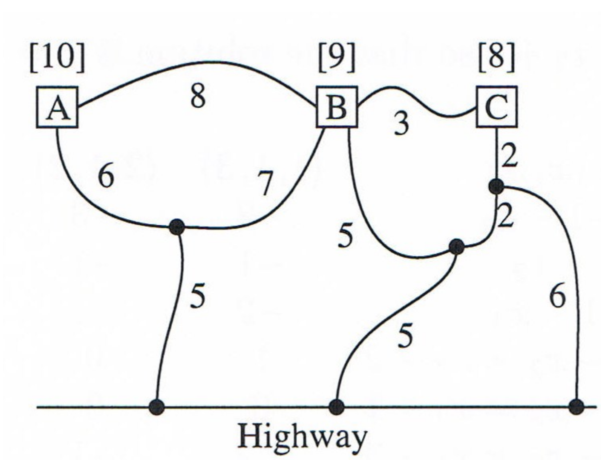

Figura 1:

If you are not entitled for continuous evaluation (or do not wish) then you must also answer the following

7. Consider the cooperative TU bimatrix game:

$$\begin{bmatrix} (3,2) & (4,1) & (4,2) \\ (4,2) & (2,3) & (4,1) \\ (1,3) & (3,0) & (4,3) \end{bmatrix}.$$

a. Find the TU-values.

b. Find the associated side payment.

c. Find the optimal threat strategies.

8. An object is to be sold through an auction. There are two potential buyers, the players. The value of the object to player $i \in \{0,1\}$ is v_i and he knows it. Each player i submits a bid b_i in a sealed envelope, simultaneously. Then, both envelopes are opened and the player who submitted the highest bid gets the object for the lowest bid. Formally, the payoffs are

$$u_i(b_0, b_1) = \begin{cases} v_i - b_{1-i} & \text{if} \quad b_i > b_{1-i} \\ (v_i - b_i)/2 & \text{if} \quad b_i = b_{1-i} \\ 0 & \text{if} \quad b_i < b_{1-i} \end{cases}$$

for $i = 1, 2$. Find the Nash equilibrium and extend to the n-player case.