# Meios Computacionais no Ensino (M.C.E) 

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Trabalho No 4

Exercício: Realização de um trabalho escrito incorporando um tema tirado da "Casa das Ciências" cuja ilustração seja feita apenas com imagens produzidas com os softwares POLY e CINDERELLA. Deve ter um mínimo de 2 ilustrações.

O trabalho deve ser carregado na página do Moodle da disciplina.

Choosed file: Demo_T_Pitagoras

The file consists of a Geogebra file that shows a right triangle and the squares of the different sides of the triangle with a number, that describes the area of the squares. One can move the points of the triangle and see that the Pythagorean Theorem is working on every right triangle. The file has also a worksheet that describes the right triangles, defines the names of the sides and explores the relations of the different sides to each other through a table. I will give an introduction of the theorem, give an interesting proof and have a look of the relationship between this right triangles and solids.

## The Pythagorean Theorem

The well-known Pythagorean Theorem is a mathematical theorem about the relations of the sides in a right triangle. It says, that in every right triangle the square of the opposite site of
the right angle, called hypotenuse, is equal to the sum of the squares of the two other sides of the triangle, called legs.

The following picture shows an example of the theorem.


The fully moveable version can be found here.
With the usual notation the hypotenuse is named $\mathbf{c}$ and the legs $\mathbf{a}$ and $\mathbf{b}$, so that the theorem can be shorten: $\mathbf{c}^{\mathbf{2}}=\mathbf{a}^{\mathbf{2}}+\mathbf{b}^{\mathbf{2}}$.

The earliest notations of this theorem can be found on a Babylonian cuneiform tablet which is dated between 1829 and 1530 AC. The influence of the great Greek mathematician Pythagoras is unknown and a subject of a big discussion between historians.

I now want to give a proof of this theorem. Although there are hundreds of proofs using different mathematical areas, I'm showing a very simple and directly understandable geometric proof.

By arranging four right triangles in a square with the length of the sides $\mathbf{a}+\mathbf{b}$, we can directly see, that the square $\mathbf{c}^{\wedge} \mathbf{2}$ has the same area then the sum two areas $\mathbf{a}^{\wedge} \mathbf{2}$ and $\mathbf{b}^{\wedge} \mathbf{2}$.


## Right triangles and polygons

Sides of polygons, that have the shape of a triangle, are mostly equilateral triangles. But let us try to find right triangles in polygons ether way. Let us have a look at the Tetrakis hexahedron, which is a Catalan Solid:


It is built from 24 isosceles triangles and these triangles in the picture seem to be right triangles. Can you imagine why this can't be true and the Tetrakis Hexahedron does not consist of right triangles

Answer: The Tetrakis hexahedron consists of 6 pyramids on a quadratic base. To build a pyramid, the sum of the angles on the top of the pyramid has to be less than $360^{\circ}$. In the case of the sum of $360^{\circ}$ the height of the pyramid is 0 and the solid would be a hexahedron, or simple: Cube.

But it is possible to find right triangles in a hexahedron. Every corner is part of a pyramid made of three right triangles on an isosceles triangle base.


I found no other solid that consists of right triangles and is, because of that, interesting for this paper. The results of the Pythagorean Theorem are filling books and are not relevant.

