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Quadratic Lie superalgebras

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By a quadratic Lie superalgebra we mean a Lie superalgebra $\mathfrak{g}$ carrying a bilinear form B on $\mathfrak{g}$ such that B is non-degenerate, supersymmetric, even, and $\mathfrak{g}$-invariant. In this case, B is called an invariant scalar product on $\mathfrak{g}$.

S. Benayadi presented an inductive description of quadratic Lie superalgebras with reductive even part and the action of the even part on the odd part completely reducible, using a particular type of double extension, namely elementary double extension [4].

We improve the result by dropping the condition of the action completely reducible. In [1], to describe inductively quadratic Lie superalgebras with even part a reductive Lie algebra, we have to use the concept of double extension of quadratic Lie superalgebras (introduced in [3]) and the notion of generalized double extension of quadratic Lie superalgebras given in [2]. Our main result says that a quadratic Lie superalgebra with a reductive even part is either $\{0\}$, basic classical Lie superalgebras and one-dimensional Lie algebra, or obtained from a finite number of previous elements by a finite sequence of double extensions by the one-dimensional Lie algebra, and/or generalized double extensions by the one-dimensional Lie superalgebra, and/or by orthogonal direct sums.

Keywords: Lie superalgebras, reductive Lie algebras, quadratic forms

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