Quadratic Lie superalgebras

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A. Medina and Ph. Revoy, 1985

- Inductive description of quadratic Lie algebras
- Double extension of quadratic Lie algebras

Examples of quadratic Lie superalgebras:

- semisimple Lie algebras
- Basic classical simple Lie superalgebras (V. Kac, 1977)
- Some solvable Lie superalgebras (A. Medina and Ph. Revoy, 1985)

A. Elduque, 1978

- structure of Lie superalgebras with reductive even part and action of the even part on the odd part is completely reducible

What about the structure of quadratic Lie superalgebras?

H. Benamor and S. Benayadi, 1999

- Double extension of quadratic Lie superalgebras

S. Benayadi, 2000

- Inductive description of quadratic Lie superalgebras such that the even part is reductive Lie algebra and the action of the even part on the odd part is completely reducible (using double extension of quadratic Lie superalgebras)

H. Albuquerque, E. Barreiro, and S. Benayadi, 2007

- Inductive description of quadratic Lie superalgebras such that the even part is reductive Lie algebra

I. Bajo, S. Benayadi and M. Bordemann

- Generalized double extension of quadratic Lie superalgebras (introduced to give the inductive description of solvable quadratic Lie superalgebras)

Lie superalgebras

A superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$
($\mathbb{Z}_2$-graded algebra $[\mathfrak{g}_\alpha, \mathfrak{g}_\beta] \subseteq \mathfrak{g}_{\alpha+\beta}, \forall\, \alpha, \beta \in \{\bar{0}, \bar{1}\}$)
is Lie superalgebra if multiplication $[,] : \mathfrak{g} \times \mathfrak{g} \longrightarrow \mathfrak{g}$ satisfies

- graded skew-symmetry property: $\forall_{X \in \mathfrak{g}_x, Y \in \mathfrak{g}_y}$
  $[X, Y] = -(-1)^{xy}[Y, X]$;
- graded Jacobi identity: $\forall_{X \in \mathfrak{g}_x, Y \in \mathfrak{g}_y, Z \in \mathfrak{g}_z}$
  $(-1)^{xz}[X, [Y, Z]] + (-1)^{xy}[Y, [Z, X]] + (-1)^{yz}[Z, [X, Y]] = 0.$

$\rightsquigarrow$ Finite dimensional Lie superalgebras over a commutative field $\mathbb{K}$ of zero characteristic

$\rightsquigarrow$ subalgebra $\mathfrak{g}_{\bar{0}}$ is a Lie algebra

$\rightsquigarrow$ vector subspace $\mathfrak{g}_{\bar{1}}$ is a $\mathfrak{g}_{\bar{0}}$-module

Quadratic Lie superalgebras

Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$

A bilinear form $B : \mathfrak{g} \times \mathfrak{g} \longrightarrow \mathbb{K}$ is:

- $\rightsquigarrow$ supersymmetric if $B(X, Y) = (-1)^{xy} B(Y, X), \ \forall_{X \in \mathfrak{g}_x, Y \in \mathfrak{g}_y}$
- $\rightsquigarrow$ non-degenerate if $X \in \mathfrak{g}$ satisfies $B(X, Y) = 0, \ \forall_{Y \in \mathfrak{g}}$, then $X = 0$.
- $\rightsquigarrow$ invariant if $B([X, Y], Z) = B(X, [Y, Z]), \ \forall_{X,Y,Z \in \mathfrak{g}}$.
- $\rightsquigarrow$ even if $B(\mathfrak{g}_{\bar{0}}, \mathfrak{g}_{\bar{1}}) = 0$.

- $\mathfrak{g}$ is quadratic Lie superalgebra if there exists a bilinear form B on $\mathfrak{g}$ such that B is even , supersymmetric, non-degenerate, and invariant.
- B is an invariant scalar product on $\mathfrak{g}$

Lie algebra $\mathfrak{g}$

center: $\mathfrak{z}(\mathfrak{g}) = \{X \in \mathfrak{g} : [X,Y] = 0, \; \forall_{Y \in \mathfrak{g}}\}$

Lie algebra $\mathfrak{g}$ is reductive if $\mathfrak{g} = s \oplus \mathfrak{z}(\mathfrak{g})$,
s is the greatest semisimple ideal of $\mathfrak{g}$.

A $\mathfrak{g}$-module V is

- irreducible if it has only two $\mathfrak{g}$-submodules (zero and itself).
- completely reducible if every $\mathfrak{g}$-submodule W of V has a *complement* W'
  (W' is a $\mathfrak{g}$-submodule of V: $V = W \oplus W'$)

Characterization of quadratic Lie superalgebras

Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$

Proposition [H. Benamor and S. Benayadi, 1999]

A bilinear form B is an invariant scalar product on $\mathfrak{g}$ if and only if the Lie algebra $(\mathfrak{g}_{\bar{0}}, B_0)$ is quadratic, where $B_0 = B \mid \mathfrak{g}_{\bar{0}} \times \mathfrak{g}_{\bar{0}}$, and on the $\mathfrak{g}_{\bar{0}}$-module $\mathfrak{g}_{\bar{1}}$ there exists a skew-symmetric non-degenerate $\mathfrak{g}_{\bar{0}}$-invariant bilinear form B_1 such that

$$B_0([X,Y],Z) \quad = \quad B_1(X,[Y,Z]), \qquad \forall_{X,Y \in \mathfrak{g}_{\bar{1}}, Z \in \mathfrak{g}_{\bar{0}}}.$$

$\rightsquigarrow$ dimension of $\mathfrak{g}_{\bar{1}}$ is even.

reduction to the B-irreducible case

$\rightsquigarrow$ $(\mathfrak{g}, B)$ quadratic Lie superalgebra

Proposition

Then $\mathfrak{g} = \bigoplus_{k=1}^{m} \mathfrak{g}_k$, where

- $\mathfrak{g}_k$ are non-degenerate graded ideals of $\mathfrak{g}$
- $(\mathfrak{g}_k, B_k = B\mid_{(\mathfrak{g}_k \times \mathfrak{g}_k)})$ are B_k-irreducible graded ideals of $\mathfrak{g}$
- $B(\mathfrak{g}_k, \mathfrak{g}_{k'}) = \{0\}$, $\forall k, k' \in \{1, \ldots, m\}$ $(k \neq k')$

Double extension

- $(\mathfrak{g}, B)$ quadratic Lie superalgebra
- $\mathfrak{h}$ Lie superalgebra
- $\psi : \mathfrak{h} \longrightarrow \mathrm{Der}_a(\mathfrak{g}, B)$ homomorphism of Lie superalgebras
- $\varphi : \mathfrak{g} \times \mathfrak{g} \longrightarrow \mathfrak{h}^*$

$$\varphi(X,Y)(Z) = (-1)^{(x+y)z} B(\psi(Z)(X), Y), \ \forall_{X \in \mathfrak{g}_x, Y \in \mathfrak{g}_y, Z \in \mathfrak{h}_z}$$

Proposition

$\mathfrak{g} \oplus \mathfrak{h}^*$

$$[X+f, Y+h] \ = \ [X,Y]_{\mathfrak{g}} + \varphi(X,Y), \ \forall_{(X+f),(Y+h) \in (\mathfrak{g} \oplus \mathfrak{h}^*)}$$

is Lie superalgebra.

$\mathfrak{g} \oplus \mathfrak{h}^* \rightsquigarrow$ central extension of $\mathfrak{g}$ by $\mathfrak{h}^*$ (by means of φ)

Theorem

- $\mathfrak{k} = \mathfrak{h} \oplus \mathfrak{g} \oplus \mathfrak{h}^*$

$$[Z + X + f, W + Y + h] = [Z, W]_{\mathfrak{h}} + [X, Y]_{\mathfrak{g}} + \psi(Z)(Y)$$
$$-(-1)^{xy}\psi(W)(X) + \pi(Z)(h) - (-1)^{xy}\pi(W)(f) + \varphi(X, Y),$$

$\forall_{(Z+X+f)\in\mathfrak{k}_x,(W+Y+g)\in\mathfrak{k}_y}$, is Lie superalgebra.

$\rightsquigarrow$ If γ is supersymmetric invariant bilinear form on $\mathfrak{h}$

- $\widetilde{B} : \mathfrak{k} \times \mathfrak{k} \longrightarrow \mathbb{K}$

$$\widetilde{B}(Z + X + f, W + Y + h) = B(X, Y) + \gamma(Z, W)$$
$$+f(W) + (-1)^{xy}h(Z), \ \ \forall_{(Z+X+f)\in\mathfrak{k}_x,(W+Y+h)\in\mathfrak{k}_y}$$

is invariant scalar product on $\mathfrak{k}$.

$(\mathfrak{k}, \widetilde{B}) \rightsquigarrow$ double extension of $(\mathfrak{g}, B)$ by $\mathfrak{h}$ (by means of ψ).

Conversely

- $\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ is a B-irreducible quadratic Lie superalgebra
- $\rightsquigarrow$ $\mathfrak{z}(\mathfrak{g}) \cap \mathfrak{g}_{\bar{0}} \neq \{0\}$
- $\rightsquigarrow$ $\dim \mathfrak{g} > 1$

Proposition

Then $(\mathfrak{g}, B)$ is a double extension of a quadratic Lie superalgebra $(\mathfrak{h}, \widetilde{B})$ $(\dim \mathfrak{h} = \dim \mathfrak{g} - 2)$ by a one-dimensional Lie algebra.

Sufficient condition

- $(\mathfrak{g}, B)$ is a B-irreducible quadratic Lie superalgebra
- Sufficient condition: I maximal graded ideal of $\mathfrak{g}$, $\mathfrak{g} = I \oplus V$, V is a Lie subsuperalgebra of $\mathfrak{g}$

Proposition

Then $(\mathfrak{g}, B)$ is a double extension of the quadratic Lie superalgebra $(I/I^{\perp}, \widetilde{B})$ by V, $\widetilde{B}$ invariant scalar product on $I/I^{\perp}$ induced by B.

$\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra

$\rightsquigarrow$ $\mathfrak{g}_{\bar{1}}$ is $\mathfrak{g}_{\bar{0}}$-module completely reducible

$\rightsquigarrow$ $\mathfrak{g}_{\bar{1}} \neq \{0\}$

Lemma

Then $\mathfrak{g}_{\bar{1}} = \displaystyle\bigoplus_{i=1}^{n} U_i$, where

- U_i is a $\mathfrak{g}_{\bar{0}}$-submodule of $\mathfrak{g}_{\bar{1}}$ such that $B\mid_{(U_i \times U_i)}$ is non-degenerate, $\forall i \in \{1, \ldots, n\}$
- U_i is irreducible or $U_i = U_{i1} \oplus U_{i2}$, where U_{i1} and U_{i2} are irreducible $\mathfrak{g}_{\bar{0}}$-submodule of $\mathfrak{g}_{\bar{1}}$ such that $B(U_{i1}, U_{i1}) = \{0\}$, $B(U_{i2}, U_{i2}) = \{0\}$, $\forall i \in \{1, \ldots, n\}$
- $B(U_i, U_j) = \{0\}$, $\forall i, j \in \{1, \ldots, n\}$ $(i \neq j)$

Elementary double extension (by the one-dimensional Lie algebra $\mathbb{K}e$)

- $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra
- $\mathfrak{g}_{\bar{1}}$ is $\mathfrak{g}_{\bar{0}}$-module completely reducible
- $D : \mathfrak{g} \longrightarrow \mathfrak{g}$ homogeneous superantisymmetric superderivation of degree 0 of $\mathfrak{g}$

  (i) $D(\mathfrak{g}_{\bar{0}}) = D(\bigoplus_{i=1}^{m} U_i) = 0$

  $(U_i$ irreducible $\mathfrak{g}_{\bar{0}}$-submodule of $\mathfrak{g}_{\bar{1}}$, $\forall i \in \{1, \ldots, m\})$

  (ii) $D\,|_{U_{i1}} = k_i Id_{U_{i1}}$, $D\,|_{U_{i2}} = -k_i Id_{U_{i2}}$, $k_i \in \mathbb{K}$, $i \in \{m+1, \ldots, n\}$

- $\psi : \mathbb{K}e \longrightarrow \mathrm{Der}_a(\mathfrak{g}, B)$ homomorphism of Lie superalgebras: $\psi(ke) = kD$, $k \in \mathbb{K}$
- $(\mathfrak{k}, \widetilde{B})$ double extension of $(\mathfrak{g}, B)$ by $\mathbb{K}e$ (by means of ψ)

$(\mathfrak{k}, \widetilde{B}) \rightsquigarrow$ elementary double extension of $(\mathfrak{g}, B)$ by the one-dimensional Lie algebra.

Quadratic Lie superalgebras: reductive even part; action of the even part on the odd part completely reducible(S. Benayadi 2000)

Inductive description quadratic Lie superalgebras:

- reductive even part
- action of the even part on the odd part is completely reducible

$\Uparrow$

Double extension

Examples

Example [V. Kac, 1977]

Classical simple Lie superalgebras $A(m,n)$, $B(m,n)$, $D(m,n)$, $C(n)$, $D(2,1,\alpha)$, $F(4)$ and $G(3)$.

Example

$\mathfrak{M} = \langle e_1, e_2 \rangle$ two-dimensional abelian Lie superalgebra with zero even part.
Invariant scalar product on $\mathfrak{M}$: bilinear form $B : \mathfrak{M} \times \mathfrak{M} \longrightarrow \mathbb{K}$
$B(e_1,e_1) = B(e_2,e_2) = 0$ and $B(e_1,e_2) = -B(e_2,e_1) = 1$.

$\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ is B-irreducible quadratic Lie superalgebra

Lemma

- $\rightsquigarrow$ $\mathfrak{g}_{\bar{1}}$ is $\mathfrak{g}_{\bar{0}}$-module completely reducible
- $\rightsquigarrow$ $\mathfrak{g}_{\bar{0}} \neq 0$

Then $[\mathfrak{g}_{\bar{0}}, \mathfrak{g}_{\bar{1}}] = \mathfrak{g}_{\bar{1}}$ and $\mathfrak{z}(\mathfrak{g}) = \mathfrak{z}(\mathfrak{g}) \cap \mathfrak{g}_{\bar{0}}$.

Theorem

- $\rightsquigarrow$ $\mathfrak{g}_{\bar{0}}$ reductive Lie algebra
- $\rightsquigarrow$ $\mathfrak{g}_{\bar{1}}$ is $\mathfrak{g}_{\bar{0}}$-module completely reducible

Then $\mathfrak{g}$ is simple if and only if $\mathfrak{z}(\mathfrak{g}) = \{0\}$.

Inductive description of quadratic Lie superalgebra with reductive even part and action of the even part on the odd part is completely reducible

$$\mathfrak{V} := \left\{ \begin{array}{l} \{0\}, \text{basic classical Lie superalgebras} \\ \text{one-dimensional Lie algebra}, \mathfrak{M} \end{array} \right\}$$

$\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra

$\rightsquigarrow$ $\mathfrak{g}_{\bar{0}}$ reductive Lie algebra

$\rightsquigarrow$ $\mathfrak{g}_{\bar{1}}$ is $\mathfrak{g}_{\bar{0}}$-module completely reducible

Theorem

Then $\mathfrak{g}$ is either an element of $\mathfrak{V}$ or $\mathfrak{g}$ is obtained from a finite number of elements of $\mathfrak{V}$ by a sequence of elementary double extensions by the one-dimensional Lie algebra and/or orthogonal direct sums of quadratic Lie superalgebras from a finite number of elements of $\mathfrak{V}$.

central extension

- $\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra
- $\rightsquigarrow$ $(\mathbb{K}e)_{\bar{1}}$ one-dimensional abelian Lie superalgebra
- $\rightsquigarrow$ D odd skew-supersymmetric superderivation of $(\mathfrak{g}, B)$
- $\rightsquigarrow$ $\varphi : \mathfrak{g} \times \mathfrak{g} \longrightarrow \mathbb{K}$

$$\varphi(X, Y) = -B(D(X), Y), \ \forall_{X,Y \in \mathfrak{g}}$$

Proposition

- $\mathfrak{g} \oplus \mathbb{K}e^*$

$$[X + \alpha e^*, Y + \beta e^*] = [X, Y]_{\mathfrak{g}} + \varphi(X, Y)e^*,$$
$$\forall_{(X+\alpha e^*),(Y+\beta e^*) \in (\mathfrak{g} \oplus \mathbb{K}e^*)}$$

is Lie superalgebra.

$\mathfrak{g} \oplus \mathbb{K}e^*$ $\rightsquigarrow$ central extension of $\mathfrak{g}$ by $\mathbb{K}e^*$ (by means of φ)

Generalized semi-direct product

- $\mathfrak{g}, \mathfrak{h}$ Lie superalgebras
- $\Omega : \mathfrak{g} \longrightarrow Der(\mathfrak{h})$ linear map
- $\zeta : \mathfrak{g} \times \mathfrak{g} \longrightarrow \mathfrak{h}$ even skew-supersymmetric bilinear map such that, $\forall_{X \in \mathfrak{g}_x, Y \in \mathfrak{g}_y, Z \in \mathfrak{g}_z}$:
  - (i) $[\Omega(X), \Omega(Y)] - \Omega([X,Y]_{\mathfrak{g}}) = ad(\zeta(X,Y))$
  - (ii) $\sum_{cycl} (-1)^{xz} \Big\{ \Omega(X)(\zeta(Y,Z)) - \zeta([X,Y]_{\mathfrak{g}}, Z) \Big\} = 0$

Proposition

- $\mathfrak{g} \oplus \mathfrak{h}$

$$[X+f, Y+h] = [X,Y]_{\mathfrak{g}} + [f,h]_{\mathfrak{h}} + \Omega(X)(h) - (-1)^{xy}\Omega(Y)(f) + \zeta(X,Y),$$

$\forall_{(X+f) \in \mathfrak{k}_x, (Y+h) \in \mathfrak{k}_y}$,is Lie superalgebra.

$\mathfrak{g} \oplus \mathfrak{h} \rightsquigarrow$ generalized semi-direct product of $\mathfrak{g}$ and $\mathfrak{h}$ (by means of Ω and ζ)

Generalized double extension

- $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra
- D odd skew-supersymmetric superderivation of $(\mathfrak{g}, B)$
  $X_0 \in \mathfrak{g}_{\bar{0}}$ such that

$$D(X_0) = 0,\ B(X_0, X_0) = 0,\ D^2 = \frac{1}{2}[X_0, .]_{\mathfrak{g}}$$

- $$\begin{array}{rcl} \Omega : \mathbb{K}e & \longrightarrow & Der(\mathfrak{g} \oplus \mathbb{K}e^*) \\ e & \longrightarrow & \widetilde{D} \end{array}$$

  $\widetilde{D} : \mathfrak{g} \oplus \mathbb{K}e^* \longrightarrow \mathfrak{g} \oplus \mathbb{K}e^*$ such that $\widetilde{D}(e^*) = 0$

$$\widetilde{D}(X) = D(X) - (-1)^x B(X, X_0)e^*, \quad \forall_{X \in \mathfrak{g}_x}$$

- $\zeta : \mathbb{K}e \times \mathbb{K}e \longrightarrow \mathfrak{g} \oplus \mathbb{K}e^*$ bilinear map $\zeta(e, e) = X_0$

Generalized double extension

Proposition

- $\mathfrak{k} = \mathbb{K}e \oplus \mathfrak{g} \oplus \mathbb{K}e^*$
  $[,] : \mathfrak{k} \times \mathfrak{k} \longrightarrow \mathfrak{k}$ even skew-symmetric bilinear map such that

$$\begin{aligned} [e,e] &= X_0, \; [e^*,\mathfrak{k}] = \{0\} \\ [e,X] &= D(X) - B(X,X_0)e^*, \qquad \forall_{X\in\mathfrak{g}_x} \\ [X,Y] &= [X,Y]_{\mathfrak{g}} - B(D(X),Y)e^*, \qquad \forall_{X,Y\in\mathfrak{g}} \end{aligned}$$

  is generalized semi-direct product of $\mathfrak{g} \oplus \mathbb{K}e^*$ by the one-dimensional Lie superalgebra $(\mathbb{K}e)_{\bar{1}}$ (by means of Ω, ζ)
- supersymmetric bilinear form $\widetilde{B} : \mathfrak{k} \times \mathfrak{k} \longrightarrow \mathbb{K}$ such that

$$\begin{aligned} &\widetilde{B}\,|_{\mathfrak{g}\times\mathfrak{g}} = B, \; \widetilde{B}(e^*,e) = 1 \\ &\widetilde{B}(\mathfrak{g},e) = \widetilde{B}(\mathfrak{g},e^*) = \{0\}, \; \widetilde{B}(e,e) = \widetilde{B}(e^*,e^*) = 0 \end{aligned}$$

  is invariant scalar product on $\mathfrak{k}$

$(\mathfrak{k}, \widetilde{B}) \rightsquigarrow$ generalized double extension of $(\mathfrak{g}, B)$ by the one-dimensional Lie superalgebra $(\mathbb{K}e)_{\bar{1}}$ (by means of D and X_0)

Conversely

- $\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ is a B-irreducible quadratic Lie superalgebra
- $\rightsquigarrow$ $\mathfrak{z}(\mathfrak{g}) \cap \mathfrak{g}_{\bar{1}} \neq \{0\}$
- $\rightsquigarrow$ $\dim \mathfrak{g} > 1$

Proposition

Then $(\mathfrak{g}, B)$ is a generalized double extension of a quadratic Lie superalgebra $(\mathfrak{h}, \widetilde{B})$ ($\dim \mathfrak{h} = \dim \mathfrak{g} - 2$) by a one-dimensional Lie superalgebra with zero even part.

Generalized double extension

Inductive description
quadratic Lie superalgebras:
• reductive even part
• action of the even part on the odd part is completely reducible
[S. Benayadi]

Inductive description
quadratic Lie superalgebras:
• reductive even part

Double extension

Double extension

Quadratic Lie superalgebras with reductive even part

$\mathfrak{g}$ Lie superalgebra
$soc(\mathfrak{g})$:= sum of all minimal graded ideals of $\mathfrak{g}$

$\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra
$\rightsquigarrow$ $\mathfrak{g}_{\bar{0}}$ reductive Lie algebra

Results

- $soc(\mathfrak{g}) = s \oplus \mathfrak{z}(\mathfrak{g})$,
  s is the greatest semisimple graded ideal of $\mathfrak{g}$.
- $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ B-irreducible quadratic Lie superalgebra
  $\neq$: one-dimensional Lie algebra
  simple Lie superalgebra
  then $soc(\mathfrak{g}) = \mathfrak{z}(\mathfrak{g})$.
- $\mathfrak{g}$ is semisimple if and only if $\mathfrak{z}(\mathfrak{g}) = \{0\}$.

Inductive description of quadratic Lie superalgebra with reductive even part

$$\mathfrak{W} := \left\{ \begin{array}{l} \{0\}, \text{basic classical Lie superalgebras} \\ \text{one-dimensional Lie algebra} \end{array} \right\}$$

$\rightsquigarrow$ $(\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}, B)$ quadratic Lie superalgebra

$\rightsquigarrow$ $\mathfrak{g}_{\bar{0}}$ reductive Lie algebra

Theorem

Then $\mathfrak{g}$ is either an element of $\mathfrak{W}$ or $\mathfrak{g}$ is obtained from a finite number of elements of $\mathfrak{W}$ by a sequence of double extensions by the one-dimensional Lie algebra and/or a sequence of generalized double extensions by the one-dimensional Lie superalgebra with even part zero and/or by orthogonal direct sums.

Malcev superalgebras

H. Albuquerque and S. Benayadi, 2004

Inductive description of quadratic Malcev superalgebras such that the even part is reductive Malcev algebra and the action of the even part on the odd part is completely reducible

H. Albuquerque, E. Barreiro and S. Benayadi, 2007

Inductive description of quadratic Malcev superalgebras with reductive even part

$\rightsquigarrow$ non-Lie Malcev algebra of dimension 7