Modules with chain conditions up to isomorphism and artinian dimension

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The content of the two papers in the references will be presented. We have studied modules with chain conditions up to isomorphism, in the following sense. Let R be a ring and M be a right R-module. We say that M is *isoartinian* if, for every descending chain $M \geq M_1 \geq M_2 \geq \dots$ of submodules of M, there exists an index $n \geq 1$ such that M_n is isomorphic to M_i for every $i \geq n$. Dually, we say that M is *isonoetherian* if, for every ascending chain $M_1 \leq M_2 \leq \dots$ of submodules of M, there exists an index $n \geq 1$ such that $M_n \cong M_i$ for every $i \geq n$. Similarly, we say that M is *isosimple* if it is non-zero and every non-zero submodule of M is isomorphic to M. We study these three classes of modules, determining a number of their properties. The results we obtain give a good description of these modules and often have a surprising analogy with the "classical" case of artinian, noetherian and simple modules. For instance, we prove that any isoartinian module contains an essential submodule that is a direct sum of isosimple modules. The endomorphism ring of an isosimple module M_R is a right Ore domain E, whose principal right ideals form a noetherian modular lattice with respect to inclusion. We say that a ring R is *right isoartinian* if the right module R_R is isoartianian. Similarly for right isonoetherian rings. We define the *artinian dimension* of a module, which is an ordinal number that measures how far a module is from the class of artinian modules. The zero module is of artinian dimension 0, artinian modules are of artinian dimension 1, if a module M has artinian dimension, then its submodules have artinian dimension $\leq \text{art.dim}(M)$, and a module has artinian dimension if and only if it is isoartinian. Several results will be presented.

References

[1] A. Facchini and Z. Nazemian, *Modules with chain conditions up to isomorphism*, J. Algebra **453** (2016), 578–601.

[2] A. Facchini and Z. Nazemian, *Artinian dimension and isoradical of modules*, accepted for publication in J. Algebra (2017).

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