On flat 2-functors

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The notion of flat module has a classical generalization to set-valued functors $C \xrightarrow{P} \mathcal{E}ns$ ([1], [4]). The main theorem of that theory expresses the equivalences

i) P is flat.

ii) P is a filtered colimit of representable functors.

iii) The diagram of P is a filtered category.

For an arbitrary *base* category $\mathcal{V}$ instead of $\mathcal{E}ns$, Kelly [3] has developed a theory of flat $\mathcal{V}$-enriched functors $C \xrightarrow{P} \mathcal{V}$, but there is no known generalization of the theorem above for any $\mathcal{V}$ other than $\mathcal{E}ns$.

In [2] we have established a 2-dimensional version of this theorem, i.e. for a 2-functor $\mathcal{C} \xrightarrow{P} \mathcal{C}at$, where $\mathcal{C}$ is a 2-category and $\mathcal{C}at$ is the 2-category of categories. As it is usually the case for 2-categories, the $\mathcal{C}at$-enriched notion of limit isn't adequate for most purposes and the *relaxed* bi and pseudo notions are the important ones.

References

[1] Artin M., Grothendieck A., Verdier J., *SGA 4, Ch IV*, Springer Lecture Notes in Mathematics **269** (1972).

[2] Descotte M.E., Dubuc E., Szyld M.,*On the notion of flat 2-functor*, submitted, arXiv:1610.09429v2 (2016).

[3] Kelly G. M.,*Structures defined by finite limits in the enriched context I*, Cahiers de Topologie et Géométrie Différentielle Catégoriques **23** (1982).

[4] Mac Lane S., Moerdijk I, *Sheaves in Geometry and Logic: a First Introduction to Topos Theory*, Springer, New York (1992).

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