

On the local cartesian closure of exact completions

Jacopo Emmenegger

April 25, 2017

A recently discovered oversight in Carboni and Rosolini’s characterisation of (local) cartesian closure of exact completions entails that such characterisation is only valid in the presence of products (resp. pullbacks), and not with just weak finite limits as claimed in [1].

We will introduce a condition on a category with weak finite limits which is sufficient to obtain the local closure of its exact completion in the most general case. This condition was inspired by an axiom in the context of constructive set theory, and originally applied to obtain the local cartesian closure of a constructive version of Lawvere’s ETCS [2].

We will see however that it naturally arises in the homotopy-theoretic context as well. In particular, it can be used to conclude that the exact completion of the homotopy category of topological spaces is locally cartesian closed, thus answering a question posed in [3].

References

- [1] A. Carboni and G. Rosolini, *Locally cartesian closed exact completions*, Jour. Pure Appl. Alg. **154** (2000) 103–116.
- [2] J. Emmenegger and E. Palmgren, *Exact completion and constructive theories of sets*, in preparation.
- [3] M. Gran and E.M. Vitale, *On the exact completion of the homotopy category*, Cah. Top. Géom. Différ. Catég. **4** (1998) 287-297.