Multiplication alteration by two-cocycles. The non associative version

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Let R be a commutative ring with unit and denote the tensor product over R by $\otimes$. In [9] we can find one of the first interesting examples of multiplication alteration by a 2-cocycle for R-algebras. In this case Sweedler proved that, if U is an associative unitary R-algebra with a commutative subalgebra A and $\sigma = \sum a_i \otimes b_i \otimes c_i \in A \otimes A \otimes A$ is an Amistur 2-cocycle, then U admits a new associative an unitary product defined by $u \bullet v = \sum a_i \, ub_i \, vc_i$ for all $u, v \in U$. Later, Doi discovered in [5] a new contruction to modify the algebra structure of a bialgebra A over a field F using an invertible 2-cocycle σ in A. In this case if $\sigma : A \otimes A \to F$ is the 2-cocycle, the new product on A is defined by

$$a * b = \sum \sigma(a_1 \otimes b_1) a_2 b_2 \sigma^{-1}(a_3 \otimes b_3)$$

for $a, b \in A$. With the new algebra structure and the original coalgebra structure, A is a new bialgebra denoted by A^σ, and if A is a Hopf algebra with antipode λ_A, so is A^σ whit antipode given by

$$\lambda_{A^\sigma}(a) = \sum \sigma(a_1 \otimes \lambda_A(a_2)) \lambda_A(a_3) \sigma^{-1}(\lambda_A(a_4) \otimes a_5).$$

for $a \in A$. One of the main remarkable examples of this construction is the Drinfeld double of a Hopf algebra H. If H^* is the dual of H and we denote by A the tensor product $H^{*cop} \otimes H$, the Drinfeld double of H can be obtained as A^σ where σ is defined by $\sigma((x \otimes g) \otimes (y \otimes h)) = x(1_H) y(g) \varepsilon_H(h)$ for $x, y \in H^*$ and $h, g \in H$. As was pointed by Doi and Takeuchi in [6] "this will be the shortest description of the multiplication of the Drinfeld double".

Motivated for the recent interest in non associative Hopf algebraic structures (see [4], [8], [7], [2]), in this talk, following [3], we introduce the theory of multiplication alteration by two-cocycles for non associative structures like non associative bimonoids with left or right division. We also explore the connections between Yetter-Drinfeld modules and projections of Hopf quasigroups (see [1]), skew pairings, and quasitriangular structures obtaining, like in the classical case, a link between Yetter-Drinfeld modules and (strong) projections of Hopf quasigroups. As a consequence, our results are a generalization to the non associative setting of the main results given by Doi and Takeuchi for Hopf algebras and includes a way to obtain proper examples of braided Hopf quasigroups.

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