A categorical approach to the maximum theorem

Seerp Roald Koudenburg*

April 19, 2017

Given a relation $J \subseteq A \times B$ between topological spaces and a continuous map $d\colon B \to [0, \infty]$, consider the function $r\colon A \to [0, \infty]$ given by the suprema

$$rx = \sup d\{y \in B \mid (x, y) \in J\}.$$

Berge's maximum theorem, which is used in mathematical economics, gives conditions on the relation J ensuring the continuity of r.

Following [1], in this talk we describe the maximum theorem in terms of the theory of monoidal topology and the theory of double categories. This approach allows us to generalise the maximum theorem, which is classically stated for topological spaces, to pretopological spaces, approach spaces and probabilistic approach spaces, amongst others.

References

[1] S. R. Koudenburg, *A categorical approach to the maximum theorem*, preprint available as `arXiv:1704.00209`.

*Middle East Technical University — Northern Cyprus Campus