CONNECTIONS BETWEEN ALGEBRAIC CATEGORIES

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In this talk we give an overview of the work developed in the past three years concerning the topic of the title, one of the tasks of the project "Categorical Methods in Non Abelian Algebra". We mainly explored properties of n-permutable categories. For $n = 2$, we obtained a new homological lemma, called the Cuboid Lemma, which characterises Mal'tsev (= 2-permutable) categories. We extended this lemma to the context of 2-star-permutable categories, a generalisation of regular Mal'tsev and normal subtractive categories; the later are known to be characterised by the 3×3 Lemma. For $n = 3$, we investigated 3-star-permutability, an abstract categorical setting which leads to a unified treatment of regular subtractive categories and Goursat (= 3-permutable) categories. For arbitrary n, we extended the characterisation of n-permutable varieties of universal algebras due to J. Hagemann to regular categories: a regular category has n-permutable congruences if and only if every internal reflexive relation R in it satisfies $R^{\circ} \leq R^{n-1}$, or equivalently, $R^n \leq R^{n-1}$; these results where only known for $n = 2$. We also proved that in a regular category all reflexive and transitive relations are symmetric if and only if every internal category is an internal groupoid, conditions that hold when the category is n-permutable.

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