

V. Lax Algebras as Spaces¹

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¹In: *Monoidal Topology. A Categorical Approach to Order, Metric, and Topology*, pp. 375-465. D. Hofmann, G. Seal, W. Tholen (Eds), Encyclopedia Math. Appl. 153, Cambridge Univ. Press, 2014