

# Collaborative Aspects of the *WGL* Project

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## Abstract

The *Web Geometry Laboratory (WGL)* project's goal is, to build an adaptive and collaborative blended-learning Web-environment, integrating dynamic geometry systems (DGS) and geometry automated theorem provers (GATP). The Web-based *WGL* environment aims to become a learning environment for geometry to be used during classes and also outside the classroom.

A learning environment should be collaborative, i.e., it should allow the knowledge to emerge and appear through interaction between its users. An e-learning environment should add to that the adaptive features, allowing the system to adapt itself, giving an individualised support to each one of its users. A blended-learning environment for geometry should include intelligent geometric tools such as the DGS and the GATP.

Being a Web-environment *WGL* can be used in a classroom and also outside of a classroom. The individualised access to the platform, the creation of groups and a system of permissions, will allow to implement the adaptive and collaborative modules. Finally the inclusion of a DGS and the connection with a GATP will allow to fulfil the above stated goals.

This contribution will focus on the collaborative aspects of *WGL*. The collaborative work among students and teachers will be possible by the open channels of communication between teachers and students and between students. The exchange of information will be in text form (chats), oral (classroom interaction), but the main issue is given by the exchange of geometric information. Each student will have access to his/her DGS instance but also to the group DGS instance where the group construction is being constructed collaboratively.

The collaborative module of the *WGL* environment, allowing the exchange of geometric contents, will open new possibilities to teachers and students, creating a more dynamic learning environment and contributing in this way to the students success.

*Keywords:* Collaborative Learning Environment, Dynamic Geometry Systems, Geometry Automated Theorem Proving

## 1 Introduction

The use of computational tools in a learning environment can greatly enhance its dynamic, adaptive and collaborative features. It could also extend the learning environment from the classroom to outside of the fixed walls of the school. In the high-school curriculum in Portugal the use of such tools is praised. Quoting from the official curriculum specification<sup>1</sup> [3]:

The computer, by its own potential, namely in the areas of dynamic geometry, function representation and simulation, allows activities of exploration and research and also activities of recovery and development, in such a way that it constitutes an important asset to teachers and students, its use should be considered compulsory in this curriculum.

To build an adaptive and collaborative blended-learning environment for geometry, we claim that we should integrate DGS, GATP and repositories of geometric problems (RGP) in a Web system capable of individualised access and asynchronous and synchronous interactions. A system with that level of integration will allow building an environment where each student can have a broad experimental, but with a strong formal support, learning platform. In the next paragraphs we will speak in more detail what do we mean by each of these features.

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<sup>1</sup>Translated from the Portuguese text.

**Blended-learning Web environment** A learning environment for geometry should be, in our opinion, a blended-learning environment<sup>2</sup>. It should be an environment that can be used as a geometry laboratory in a classroom by teachers and students in a much enhanced substitute for the ruler and compass physical instruments. But it should also allow to extend itself outside the classroom, for homework tasks, for problems proposed by the teacher to be solved outside the classroom, for students tutored study where the tutors could be the teacher(s) and/or classmates, or even for the students self-study.

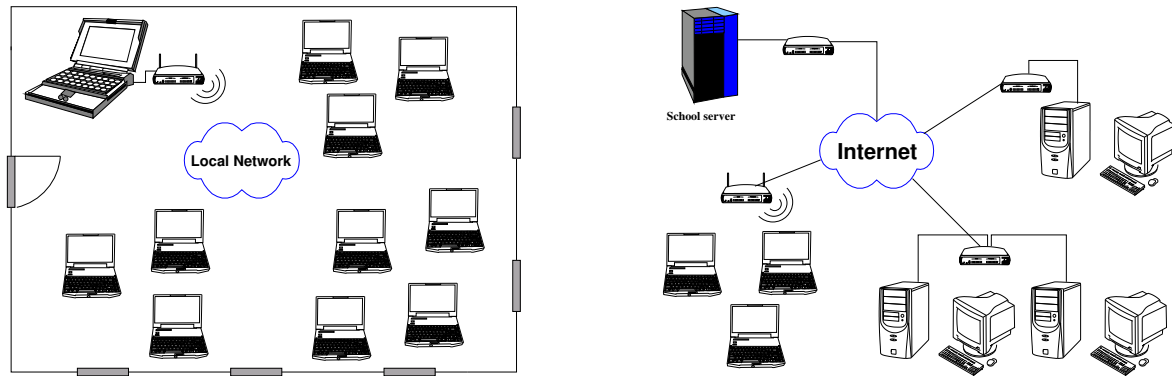


Figure 1: LAN and WAN Environments

A Web-environment is appropriate for both situations. In a classroom, a local or wide area network (LAN or WAN) environment would allow the synchronous interaction between teacher and students. Outside the classroom a WAN environment would allow synchronous and asynchronous interactions. The ubiquity of the Web nowadays make such a learning environment easily accessible (see Figure 1).

**An Adaptive Environment** The environment should be adaptive, i.e., it should adapt the help information given to different users and also, an important feature in a learning environment, to adapt the learning path to the different users needs [7, 16].

The system should be able to infer the geometric knowledge of the users or to use plan recognition, in terms of geometric knowledge, to infer the actual plan or information goal of the users [7]. We see this as an important challenge to be addressed by any new system, in *WGL* this will be realised through the registration of the information of the different actions made by the users and through the analysis of those interactions [16].

**Collaborative Environment** The environment should be collaborative, i.e., it should allow the knowledge to emerge and appear through the interaction between its users [8, 17, 19]. The teacher and students should be able to interact with each other but the environment should also be open to interaction between students. Speaking about a geometric environment this interaction should not be restricted to textual contents, (chats, wikis, etc.) but it should be extended to geometric contents. The users of such a system should be able to exchange geometric constructions, or even to build a geometric construction in a collaborative way. This exchange of geometric contents will open new possibilities in terms of a collaborative geometric learning environment. This subject will be further pursue in section 2.

**DGS Integration** The advantages of the DGS in a learning environment are multiple: they are easy to use, they stimulate the creativity and the discovery process. They provide an outstanding tool to substitute the old ruler and compass used in the classrooms. The constructions made from free objects and constructed objects allow a degree of property preserving manipulations much superior to the capabilities of physical tools.

In spite of the DGS outstanding features, they do not create a learning environment by themselves, so its integration in a learning environment will be beneficial. The DGS excel in the dynamic construction of geometric figures, the learning environment should add to this, the synchronous and asynchronous activities, the collaborative and the adaptive features.

<sup>2</sup>A blended-learning environment is a mixing of different learning environments. It combines traditional face-to-face classroom (synchronous) methods with more modern computer-mediated (asynchronous) activities.

**GATP Integration** The dynamic nature of dynamic geometry systems allow its users to manipulate the positions of the free objects in such a way that the constructed objects are also changed, yet preserving the geometric properties of the construction. These manipulations are not formal proofs, the user is considering only a finite set of concrete positions, nevertheless these tools provide a first, not yet formal, link between the theories and models of geometry.

The Pythagoras theorem is a good example where a DGS can be used in a very fruitful way. The dynamic component allows us to work the visuals proofs of this theorem. The JGEX Web page [1] has some very nice examples of visual proofs of this theorem.

Automated theorem provers are less widely used as tools in a learning environment, but geometry with its axiomatic nature is a “natural” area for a formal tool such as the GATP. The GATP give its users the possibility to reason about a given DGS construction, this is no longer a “proof by testing”, but an actual formal proof. If the GATP is capable of producing synthetic proofs, the proof itself can be an object of study, in other cases only the conclusion matters [11]. Another link between a GATP and a DGS is given by the automated deductive testing, by a GATP, of the soundness of the constructions made by a DGS [10]. In either cases (formal proofs or soundness of the constructions) we claim that the GATPs can be used in the learning process.

**Learning Environment for Geometry** As said above to have an adaptive and collaborative blended-learning environment for geometry we should integrate intelligent geometric tools in a Web system capable of asynchronous and synchronous interactions.

Such an integration it is still to be done, there are already many excellent DGS [31], some of them have some sort of integration with GATP, others with RGP [11, 22]. Some attempts to integrate these tools in a learning management system (LMS) have already been done [18, 24], but, as far as we know, all these integrations are only partial integrations. A learning environment where all these tools are integrated and can be used in a fruitful fashion does not exist yet.

In this article we focus on the collaborative aspects of the Web Geometry Laboratory (*WGL*) an Web environment that integrates a DGS and a repository of geometric problems and it will integrate also a GATP, aiming to provide an adaptive and collaborative blended-learning environment for geometry.

**Paper overview.** In section 2 we introduce collaborative learning, explaining its goals and how to achieve them. In section 2.1 we explain the collaborative features of *WGL* relating it to other existing tools. In section 2.2 we present a case study and in section 3 we speak about the work still to be done and we draw some final conclusions.

## 2 Collaborative Learning

Based on the theoretical framework of constructivism, collaborative learning should be the product of the interaction between groups of students. A collaborative environment should allow the more skilled students help others in a joint resolution of tasks allowing, in this way, each student to improve, learning more about a particular task. Collaborative learning is an effective learning strategy. It can improve the success of learning in different subjects, students skills and levels of education, placing the students in the centre of their learning process [30].

The acquisition of knowledge by students in collaborative activities results in a process in which participation in social environments enables interaction, collaboration and evaluation activities. In fact, collaborative learning is the sharing of knowledge [26]. Collaboration between students during learning can help develop strategies and skills for problem solving [29].

The contributions from these environments have three aspects [5]:

- with the joint construction of knowledge, inspired by the influence of constructivism, the student will find answers from their own knowledge and their interaction with reality and with colleagues;
- the creation of a dynamic database, worked and built by each;
- the possibility of communication between students, synchronously or asynchronously.

Learning environments supported by computer may be described in a context where the computer facilitates interaction between students to acquire knowledge, skills and attitudes [2]. There are several reasons to use this type of environments, such as:

- involves groups of students working to solve a problem or to complete a task;
- students are working in groups, mutually searching for understanding, solutions or meanings;
- is a natural social act in which students talk among themselves;
- students help and encourage each other to learn, it involves a considerable interaction.

In the process of collaborative learning, the teacher should define a plan leading to the formulation of groups of students seeking a common goal for learning. The success of these plans depends, mainly, of the frequency of the interactions among all active members [14].

The collaborative learning environments provide a place of joint knowledge construction [27] and, in the case of geometry, it should extend the group interaction in a way that the exchange of geometric information should be possible. For example, having available a task, a given geometric construction previously determined by the teacher, and communication channels between teacher/students and among groups of students, the task could be done collaboratively by the groups of students and with the help/tutoring of the teacher.

## 2.1 Web Geometry Laboratory — A Collaborative Environments for Geometry

In the case of the Web Geometry Laboratory the focus is in the exchange of geometric information, the students should be able to produce the geometric construction collaboratively.

A class session using *WGL* is understood as a Web laboratory where all the students (eventually in small groups) and the professor will have a computer running *WGL* clients. Also needed is a *WGL* server, a local, e.g. in the teacher computer, or a WAN server, e.g. in the school WAN server (see Figure 1).

The *WGL* server is the place where all the information is kept: the login information; the group definition; the geometric constructions of each user; the users activity registry; etc. In the *WGL* server is also kept the DGS applet and the GATP will also execute there. The clients will have an instance of the DGS applet each, using the server to all the needed information exchange.

After installing a *WGL* server the administrator of the system should define all the teachers that will be using the system. The teachers will be privileged users in the sense that they will be capable of define other users, their students. In the beginning of each school year the teachers will define all his/her students as regular users of the *WGL*. The teacher may also define groups of users (students), these groups can be define at any given time, e.g. for a specific class, and it will be within this groups that the collaboration between its members will be possible. The definition of the groups and the membership relation between groups and its members will be the responsibility of the teachers that could create groups, delete groups and/or modify the membership relation at any given time.

Each user will have a “space” in the server where he/she can keep all the geometric construction he/she produces. Each user will have full control over this personal scrapbook, having the possibility of saving, modifying and deleting each and every construction he/she produces using the DGS applet.

To allow the work in the the geometric constructions, a permissions system will be implemented. This system will be similar to the “traditional Unix permissions” system [20]. The users will own the geometric construction defining the seeing, reading and writing permissions per geometric construction. For example, given a geometric construction `geo01`, created by the user `studentN`, belonging to the group `groupX`, the construction could have a permission `srwsr----`, meaning that the construction’s owner, the first three positions, will have all the permissions, the members of the group, the middle three positions, will be able to see and read, but not to write (modify) and all the other users, the three last positions, will have no permissions over this construction, they will even not be aware that the geometric construction exist. By default, the teacher will own all the groups he/she had created.

Planning a class the teacher has to decide the groups and the students membership. This could mean to keep the groups already working together, to create new groups, or to change the memberships relations on the existing groups. Another task could be the preparation of a set of geometric constructions: the starting points for tasks to be completed during the class; illustrative cases; etc. Manipulating the permissions, these constructions can became available to the students during the class, at the correct moment. The permissions systems and the definition of groups will allow to set different tasks for different groups of students.

The default setting will be `srwsrw---`, meaning that the creator (owner) and his/her group will have all the permissions and the others will have none. This default behaviour could be changed by the teachers.

Each *WGL* user has access to his/her constructions (see the “Scrapbook” bar in Figure 2) and, accordingly to the permissions, to the constructions of all the other users (see the “Geometric Construction List” bar in Figure 2). At any given moment he/she can download (read) the construction into the DGS, modify it and, eventually, upload the modified version into the database. By default the teacher will own all the groups he/she created, this grants to teachers access to all the constructions made by the students. It will be possible to the student to block this access setting the access to a given construction to `srw-----`, i.e., the owner of the construction is the only user that can see, read and write the construction.

Given these settings the teacher can plan a task. At a given moment, during the class, he/she “releases”, that is, make it visible, a construction, or a set of constructions, establishing a task to be performed in the *WGL* workbench. Each student can then download the construction to his/her personal work-space and begin working on it. If planned to be made collaboratively (after all that is the goal of this module in *WGL*) the student should be in a previously defined group, so each student in the group could exchange geometric information with the other members of the group.

The collaborative module of *WGL* will open a second DGS window where each student will be able to see (not to directly modify) the construction being made. The interaction between the two DGS windows will allow to export the student contribution to the group construction and to import the group construction to the student work-space. To allow to distinguish all the contributions a different colour will be attributed to each student. A textual, chat-like, line of communication will help the collaborative effort during a class, the oral communication between students will be possible by the normal classroom interaction. The teacher could always participate in this effort, this is made possible by the fact that he/she owns all the groups he/she has created.

The *WGL* collaborative features are tough mostly for a blended-learning setting, that is, a classroom/laboratory where the computer-mediated activities are combined with a face-to-face classroom interaction. Nevertheless given the fact that the *WGL* is a Web application the collaborative work can extend itself to the outside of the classroom and be used to develop collaborative work at home, e.g. solving a given homework. In this setting the only drawback it will be a slow connection to the *WGL* server. We estimate that a normal bandwidth ( $\geq 20\text{Mb}$ ) will be enough.

The *WGL* as a Web client/server application; the database (to keep: constructions; users information, constructions permissions, user’s logs); the DGS applet; the GATP and the synchronous and asynchronous interaction are all implemented using free cross-platform software, namely PHP, Javascript, Java, AJAX and MySQL. and also Web-standards like XHTML, CSS style-sheets and XML. All this will allow to build a collaborative learning environment where the capabilities of a tool like a DGS can be used in a more rich setting than in an isolated environment where (eventually) every student could have a computer with a DGS but where the communication between them would be non-existent. The exchange of text, oral and geometric information between members of a group will enrich the learning environment.

Learning environments supported by computer are seen as an important mean for non-presential education. The DGS are also important in presential environments, as a much enhanced substitute for the ruler and compass physical instruments, allowing the development of experiments, stimulating learning by experience. There are several DGS available, such as: *GeoGebra* [6], *Cinderella* [23], *Geometric Supposer* [25], *GeometerSketchpad* [9], *CaR* [4], *Cabri* [15], *GCLC* [12] but none of them defines a Web learning environment with collaborative features.

The program *Tabulae* is a DGS with Web access and with collaborative features [18, 28]. This system is close to *WGL* the permissions system and the fact that the DGS is not “hardwired” to the system but it is an external tool incorporated into the system, are features that distinguish positively *WGL* from *Tabulae*. The adaptive features, the connection to the GATP and the internationalisation/localisation are also features missing in *Tabulae*.

## 2.2 *WGL* in the Classroom

Geometry is important as a means for describing, understanding and interacting with the space we live in, being seen as the most intuitive part of mathematics, with a strong relation with the day-to-day activities. The dynamic geometry software are a very important tool in the study of geometry allowing the establishment of links between a formal view of geometry, the formal specification of a geometric construction by means of a constructive geometry language, and an intuitive view, the rendering of the geometric construction in the Cartesian plane. The DGS add to the rendering of the construction the dynamics, the possibility of moving the free objects, given in this way a first glimpse of the geometric properties possessed by a given geometric construction.

The distinct feature of a DGS over a drawing software is the possibility of constructing a geometric figure from a set of initial free points and a set of basic construction steps, that is, DGS are based on a given formal constructive geometric language, even if this language is somehow hidden behind a graphical user interface. The free points can be dynamically

moved, they are free, the other elements of the geometric construction are fixed, they cannot be moved, but whenever the free points are moved the construction moves as a whole, in such a way that the geometric properties of the construction are preserved.

A teacher designing a class plan where a DGS is to be used must consider its distinct features. The construction should not be a set of points and lines placed more or less randomly in a given plane but, as said above, should be a set of randomly chosen points, the free points, and then a set of geometric constructive steps that allow us to build the final geometric construction.

Let explore the following geometric theorem as a base for an example of a possible class plan using *WGL*.

**Theorem 1 (Circumcentre Theorem)** *The vertices of a triangle are equidistant from the circumcentre.*

To prove this we must show that, for any given triangle  $\triangle ABC$  and the perpendicular bisectors of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{AC}$ , the perpendicular bisectors intersect in a point and that point is equidistant from the vertices.

The geometric concepts behind this theorem are: triangle; midpoint of a segment; perpendicular bisector; intersection of points; equidistant points; circle. In the next section we explore the use of *WGL* for the study of this geometric theorem.

**Circumcenter Theorem Class Plan** Admitting that the concepts of triangle and circle where already being given and that we want to explore the concepts of midpoint of a segment, perpendicular bisector and equidistant points, then the starting point is the triangle.

The teacher could begin by providing the starting triangle formed by three (free) points and the lines (segments) joining them. This could be done by a previously constructed geometric figure and its “release”, i.e., the changing of its permissions (see Section 2.1), at the task start (see Figure 2).

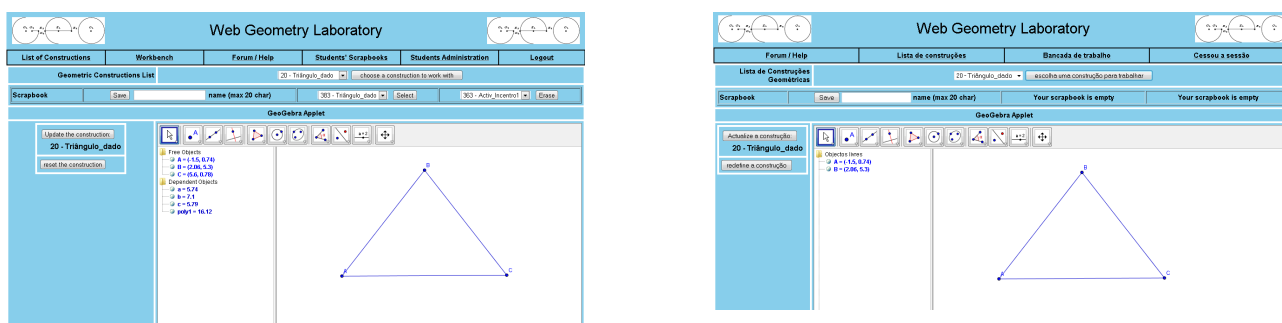


Figure 2: Teachers and Students Views

We shall now describe a class session with the student’s steps needed to complete the construction. Those steps will lead the students to the solution of the theorem described in figure 5:

1. choose the teacher’s triangle construction;
2. find the midpoints of the segments  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{AC}$ ;
3. find the bisectors of all sides of the triangle (using the previously found midpoints);
4. find the intersection of the perpendicular bisectors;
5. build the circle with centre in the point found in the previous step and going through point  $A$  (or  $B$  or  $C$ );
6. using the dynamic feature of the DGS move point  $A$  (or  $B$  or  $C$ ) to, informally, conclude that the property stated in the theorem should be true.

**Teacher-Student Collaboration** The collaboration between student and teacher plays a critical role in helping students reflect and participate in their own learning experiences. Learning should be a meaningful, active and constructive process [13].

The WGL platform allow the teacher to see what each of the student is doing, i.e., the teacher has access to the student's workbench, in this way the teacher can mediating learning through collaboration with a additional comments at any given moment and this way can help students. This comments can be textual (chat like), oral (if we are in a classroom environment), or even geometric, for example providing a new construction with some additional elements.

The first task set by the teacher should be the construction of the intersection of the perpendicular bisectors (steps 1 to 4). At this point the teacher would release the first construction.

**Student-Student Collaboration** The collaborative learning strategies for problem solving allows the building of the social skills that students need to succeed, when working in groups. These skills are important in preparing students for a world where the collaborative writing and resolution of tasks are key elements.

The second to fourth tasks are appropriated to be tackled by groups of students. Having defined the groups, the teacher will also define the student that will took the lead at the start of the task, this will be the student that will have the group's construction in his/her DGS workbench (see Figure 3).

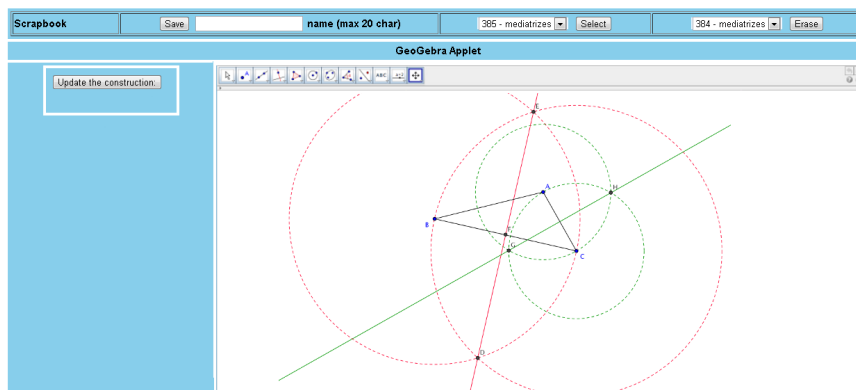


Figure 3: Collaborative Work - Student working in the group's construction

The lead student will be working in the group's construction establishing a lock in the construction. All his/her work will be broadcast to the other members of the group allowing them to follow what is being done (see Figures 3 and 4). At any given moment the lead student can release the lock, answering to another member request or by his/her own initiative, at that moment the group's construction will be copied to his/her own workbench and another member of the group will took the lead.

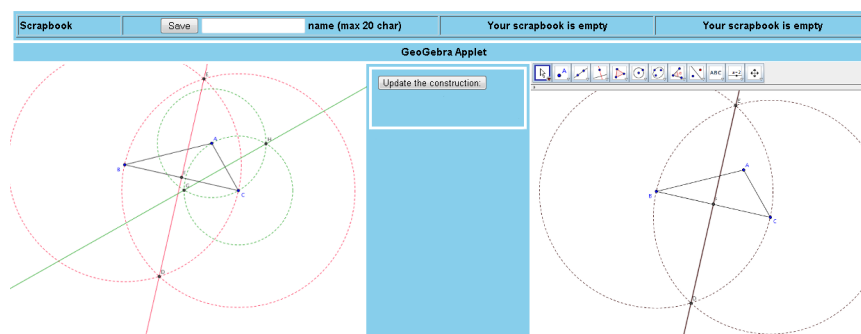


Figure 4: Collaborative Work - Student following the work being done in the group's construction

The members of the group that are not leading the group will have two distinct views over the construction. They are able to see, without being able to modify it, the work that is being done in the group's construction (window at the left in

figure 4). They will also have a personal copy of the construction (window at the right in figure 4), where they can work trying to solve the current task. When the lock in the group's construction is released the student that have asked for it will become the group leader, his/her construction will be automatically saved to his/her personal scrapbook and he/she can begin working in the group's construction (see Figure 3).

The exchange of roles between the group members and the possibility that each student has of working in his/her solution to the task, seeing at the same time what is being done by a colleague, will allow the collaborative solving of the tasks given by the teacher.

The teacher is able to see the evolution of the construction made by the groups and at any given moment the teacher can help one or more groups with textual comments. After this step is completed by the majority of the groups (if not all) the teacher could decide to adopt one of the groups' constructions as the class representative, broadcasting it to all the students. This will end the first task and a new task could then begin.

The last task is to find the set of points that are equidistant from the point found at the end of the previous task. This could lead to an introduction, made by the teacher, about equidistant points and the property that the points of belonging to a circle have related to the centre of the circle.

Adding the circumscribed circle is a relatively easy task (see Figure 5) after which the students, using the dynamic features of the DGS could convince him/herself that this is a property of this construction and that the geometric theorem should be true.

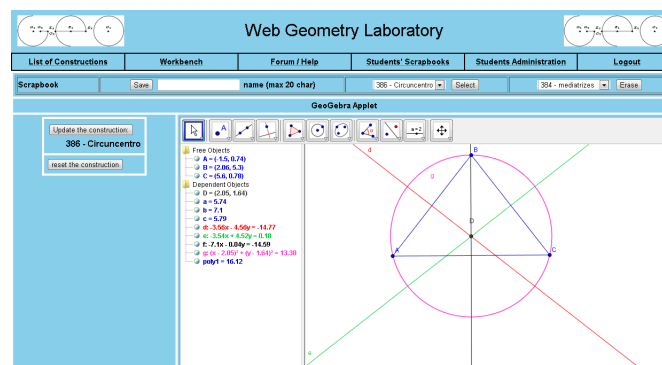


Figure 5: Final Construction

This last step should be carefully addressed by the teacher. The teacher should stress that this is not a proof of the theorem, but only a "strong evidence" that the theorem should be true. If appropriated a next task could be a formal proof of the theorem, eventually using a GATP.

For this case the teacher could also use the DGS to show that, using the degenerate case of  $A$ ,  $B$  and  $C$  being collinear points, this result is not true for all the sets of three points.

### 3 Conclusion & Future Work

When we consider a computer system for an educational setting, we feel that a blended-learning environment is the most interesting solution. That leads to a Web system capable of being used in the classroom but also outside the classroom. A learning environment should be collaborative, in geometry that means: textual; oral; but also geometric information should be able to flow among students and teachers. An e-learning environment should be adaptive, it should be able to help the student even outside the classroom, when the interaction with other could be asynchronous. The formal nature of geometry leads to tools like the GATP, not only to prove a given conjecture, but also to validate constructions and provide proofs, geometric proofs, of all the reasoning made by the prover. Finally a computer system should be modular allowing the use of the best tools, DGS and GATP available, providing a system that can be easily updated.

The *WGL* systems aims to fulfil those characteristics.

- A client/server system that can be used in a LAN or in a WAN setting.
- Modular: incorporating DGS and GATP.



- Collaborative.
- Adaptive.

The *WGL* system is a work-on-progress system. It is a client/server modular system. It has already incorporated a DGS; some adaptive features, i.e., the individualised scrapbook where all the users can keep their own constructions; some collaborative features, i.e., the teacher can see all the work done by the students and broadcast help if needed. Given the fact that it is a client/server system the incorporation of a GATP (on the server) it is not difficult. One of the authors has already experience on that type of integration [10, 21, 22].

The group management and the permissions system is already implemented. The collaborative module will be completed shortly. A first case study, involving high-school students and focusing in the use of *WGL* in a classroom, is already being prepared and it will be implemented in the spring term.

The next task will be the adaptive module, the logging of all the steps made by students and teacher and the construction of student's profiles on top of that. The last task will be the integration of the GATP in the *WGL*. We hope that at the end the *WGL* can become a very challenging learning environment for geometry.

A prototype of the *WGL* system, still without the collaborative module, is available at <http://hilbert.mat.uc.pt/WebGeometryLab/>. As soon as the collaborative module is completed it will be transferred from the development server to this (production) server.

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