Categorical algebra

Semi-abelian categories, semi-localizations and factorization systems

Marino Gran

Semi-abelian categories [9] provide a suitable context to study the (co)homology of non-abelian algebraic structures (such as groups, Lie algebras, compact groups, crossed modules, cocommutative Hopf algebras), torsion theories, and commutators. In this mini-course a brief introduction to some elementary properties of these categories will be given, such as the validity of the classical homological lemmas and the isomorphism theorems from group theory [1]. In the last part we shall focus on more advanced aspects of the theory.

These latter aspects will concern the relationship between torsion theories in semi-abelian categories and semi-left-exact reflections in the sense of [5], and the correspondence with radicals, closure operators and factorizations systems [2, 4, 6]. An abstract characterization of semi-localizations of semi-abelian categories will be given [8], extending a known result in the abelian context [10]. If time permits the notion of action representable category [3] will be briefly introduced, and illustrated by some examples in the categories of groups, Lie algebras and crossed modules.

Concerning the preliminaries needed to follow these lectures, some knowledge of the basic properties of regular categories will be helpful. These preliminaries will be briefly recalled at the beginning of the course and some notes will be available [7].

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