Symbolic computation in partial differential equations

Diogo Gomes

This course is an introduction to symbolic computation in Mathematica with an emphasis on applications in partial differential equations. We begin by discussing the Wolfram programming language, its functional programming features, and pattern matching. As a first application, we develop tools to manipulate differential operators and performing routine time-consuming tasks - changes of coordinates, compute linearizations, looking for special solutions or symmetries. Next, we examine quantifier elimination techniques and present applications that range from IMO-style problems to parametric estimates in partial differential equations. Finally, we use ideas from the calculus of variations to develop algorithms to find conserved quantities in partial differential equations, compute entropies, and systematically perform simplifications using integration by parts. Some knowledge of Mathematica will be useful, but the course will be self-contained.