Generalised complex geometry

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Generalized complex structures were introduced in the early 2000s as a simultaneous generalization of complex and symplectic structures. As such they caught the attention of mathematicians and physicists working in areas related to mirror symmetry and soon enough there were good indications that these provided a good geometric framework for string theory. Mathematically, generalized complex structures display many interesting features, one of them being that they are not homogeneous: Differently from complex and symplectic manifolds that look locally like C^n and R^2n with the standard symplectic form, respectivelly, generalized complex structured admit local invariants. In fact, complex and symplectic points can co-exist in a single connected generalized complex manifold. In this minicourse we will touch upon several of the aspects mentioned above.

Day 1) Generalized complex structures and where to find them: We will introduce the double tangent bundle, which is home to generalized complex structures and several of their cousis. We will then cover their basic properties and introduce further simple generalizations (therefore making generalized generalized complex structure ;-)

Day 2) Examples. Today we will cover a series of examples, starting from complex and symplectic and moving on to more advanced constructions, including fiber sums, connected sums and blow-ups. Several examples treated are quite concrete and related to toric geometric constructions.

Day 3) T-duality: With toric models at hand we will study a baby version of mirror symmetry. First we will study T-duality without fixed points and already here observe the topology change that comes from the presence of a 3-form flux. Then we will take a look at the steps needed to extend T-duality to circle actions with fixed points.

Day 4) Deformations and local forms. Today we will delve into the deformation problem and focus on the relation between generalized complex and holomorphic Poisson structures.