Approximation properties of inhomogeneous function spaces versus Lavrentiev's phenomenon in the calculus of variations

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Abstract

Regular functions are not necessarily dense in inhomogeneous Sobolev-type functions spaces like e.g. with the variable exponent.

Typical techniques of proving regularity of minimizers to variational functionals are based on a construction of a sequence of nice solutions to auxiliary problems, that is convergent in a relevant way. If the growth of the functional is not controlled, it is possible that such a sequence does not exist. In order to make the regularity methods work the infima of the considered functional over regular functions (e.g. smooth) and over all functions for which the functional is finite have to coincide. I will explain how a relevant density of regular functions can be ensured in a unconventional spaces, when it is impossible, and what are the consequences of these phenomena. In particular, I will make use of the density result to prove existence of solutions to nonlinear PDEs in nonreflexive spaces.

The course does not require specific knowledge apart from basic understanding of analysis and PDEs.

Day 1: Introduction to Lavrentiev's phenomenon. Classical results in one dimension. Meaning of Lavrentiev's phenomenon and density results for analysis.

Day 2: Inhomogeneous N-functions. Fenchel-Young conjugate to an inhomogeneous N-function. Basic properties needed for building well-working functional setting. Doubling conditions and explanation how much it restricts applications.

Day 3: Musielak-Orlicz and Musielak-Orlicz-Sobolev spaces. Definitions. Dual and pre-dual spaces. Density results illustrated by special instances and counterexamples.

Day 4: Application of density results - existence of elliptic PDEs and the absence of Lavrentiev's phenomenon.