



# Operads and applications to deformation theory

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## Abstract

Operads are tools for encoding algebraic structures, such as associative or Lie algebras, by referring only to their operations instead of their elements. Studying operads is analogous to studying groups instead of group representations. Taking such an abstract point of view turns out to give us not only a universal but also a very efficient tool to address natural questions in homological algebra or more generally to study the homotopy theory of algebraic structures, mostly thanks to Koszul duality. In this lecture series, after introducing the machinery of algebraic operads we will proceed in the direction of applying these tools to (infinitesimal) deformation theory. A bit more concretely: Going back to the works of Kodaira–Spencer in the 50's, a classical heuristic in deformation theory asserts that the infinitesimal deformations of an algebro-geometric object over a field of characteristic zero are controlled by a dg Lie algebra. It turns out that in various settings the operadic formalism produces very explicit and computable instances of such deformation Lie algebras in a universal way. On the other hand, we will see how the operadic formalism allows us to give some algebraic problems, such as the isomorphism problem for universal enveloping algebras, a non-obvious deformation theoretic interpretation.





Day 1: Introduction to operad theory: Operads and their algebras, examples and constructions. Homotopy theory of algebras, infinitesimal and formal deformations.

Day 2: The algebraic operadists' toolkit: Koszul duality of operads, Bar-Cobar constructions, homotopy coherent structures, deformation dg Lie algebras and their Maurer-Cartan elements. Homotopy transfer theorem.

Day 3: Exponentials and the gauge group, the BCH formula. Deformation theoretical interpretation of notions such as formality and homotopy transfer. We will study a comparison between deformations of commutative algebra structures and of their underlying associative algebra structures. We will see that this is dual to the universal enveloping algebra isomorphism problem.

Day 4. Formal moduli problems: From the classical heuristic to a formal theorem by Lurie and Pridham. Operadic moduli problems: examples, results and applications. The case of positive characteristic.

