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★**Locales treated mostly in a covariant way.**

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In this book Picado and Pultr have produced a necessary complement to P. T. Johnstone's book [*Stone spaces*, Cambridge Univ. Press, Cambridge, 1982; [MR0698074 \(85f:54002\)](#)] for all people working in locales. The authors' aim is to have this text as a first introduction to point-free thinking. The text combines both frame and localic approaches to the subject. It isn't a replacement for more technical works, but it could be used as the skeleton for an introductory graduate course.

The text starts out with concise expositions of basic facts about posets, Heyting algebras, pseudocomplements and complements. The remaining part of the introductory chapter is devoted to the notions of frame and sober space.

In what follows, the notions of locale and localic map are fully developed in the canonical manner as an extension of (sober) space and continuous map. Already at this stage a very important observation is discussed: localic maps are in fact meet-preserving maps such that their left adjoints have to preserve finite meets. The remaining part of Chapter 2 tackles the notions of point and spectrum of a locale. Here localic maps send points to points. Chapter 3 addresses the standard topics of spectrum adjunction and spatiality of locales.

The text goes on with an outline of the basic structure of localic morphisms. In this framework epimorphisms and extremal monomorphisms in locales are thoroughly studied. These two classes constitute a factorization system in locales. Chapter 5 introduces the most natural notion of a subobject of a locale, a sublocale. In fact, sublocales can be represented by onto frame homomorphism, i.e., by frame congruences. A beautiful new characterization is that a subset of a locale is a sublocale iff the corresponding embedding is a localic map. Moreover, the set of all sublocales of a locale, ordered by inclusion, is a complete lattice such that its dual poset is a frame (called a full assembly). The Chapter 6, on special sublocales, reviews the notions of a point (2-element sublocale), open and closed sublocales, dense and Boolean sublocales, and complemented and spatial sublocales, arriving at the highly pleasing statement that any sublocale is a union of Boolean ones. It is also worth mentioning that, for each locale, the spectrum of its full assembly is the Skula space of its spectrum.

Chapter 7 tackles the standard notion of the image of a sublocale under a localic map, hence the characterizations of open and closed localic maps (which are defined as in classical topology) are presented. Also, the notion of the preimage of a sublocale under a localic map is introduced. In this context a localic preimage is the largest sublocale contained in the usual set-theoretic preimage of this localic map. In Chapter 8 the connections between sublocales and the corresponding frame congruences, including the way a frame congruence is generated from relations, are presented. It serves as a preparatory chapter for Chapter 9, which deals with another important

categorical construction—products of locales. Hence the authors establish the completeness and cocompleteness of the category of locales and localic maps.

Chapters 10 and 11 cover several separation axioms studied in the context of locales—regularity, fitness, subfitness and Hausdorffness. The book ends with a chapter devoted to entourages and uniformities on locales. As an application, uniformities (left, right and two-sided) on localic groups (group objects in the category of locales) are studied.

The great strength of the book is that it is the first reasonable exposition of the theory of locales where localic maps are functions in the usual sense. The style is delightful. Picado and Pultr know how to write a good mathematical book. Naturally, such a brief text can only indicate the directions of the authors' thoughts. Indeed, as was pointed out by the authors, the text misses very important questions that are truly relevant to point-free topology. But the time was ripe for such an exposition, and Picado and Pultr have succeeded in that.

All told, the reader experiences a truly splendid text that makes a lot of things much clearer, and both beginner and expert in this subject will have a lot of fun reading it.

Reviewed by *Jan Paseka*

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