

ERRATA (26/12/2011)

| Page | Line | Where is                                     | Should be                 |
|------|------|--|---------------------------|
| 126  |      | There is a mistake in the proof of Lemma 1.4 | See corrected proof below |
| 128  | -12  | V.4.8  | V.5.8                     |

**Corrected proof of Lemma VII.1.4** (page 126):

First, observe that if  $\beta$  is a limit ordinal and  $(1, c) \in \pi_1\nu_\beta(U)$  then  $(1, c) \in \nu_\gamma(U)$  for some  $\gamma < \beta$ : indeed, by compactness there is a finite  $A$  with  $\bigvee A = 1$  and  $A \times \{c\} \subseteq \bigcup\{\nu_\gamma(U) \mid \gamma \text{ non-limit, } \gamma < \beta\}$  and by IV.5.6 then  $(1, c) \in \nu_\gamma(U)$ .

Now let the statement not hold. Then there exists a  $(1, b) \in \nu_\alpha(U)$  with  $\alpha > 1$  least among such  $(1, b)$ 's. Obviously  $\alpha$  is not a limit ordinal, and by the observation  $\alpha$  is not a successor of a limit ordinal  $\beta$  either (else  $b = \bigvee B$  such that all the  $c \in B$  are in a  $\nu_\gamma(U)$  and by the minimality of  $\alpha$ ,  $(1, c) \in \pi_2\pi_1(U)$  and  $(1, b) \in \pi_2\pi_2\pi_1(U) = \pi_2\pi_1(U)$ , a contradiction). Thus,  $(1, b) \in \nu\nu(V) = \pi_2\pi_1\pi_2\pi_1(V)$  for  $V = \nu_\gamma(U)$ . Then  $b = \bigvee B$  for some  $B$  with  $\{1\} \times B \subseteq \pi_1\pi_2\pi_1(V)$ , and for each  $c \in B$ ,  $(1, c) \in \pi_1\pi_2\pi_1(V)$  and we have  $A_y$  such that  $\bigvee A_y = 1$  and  $A_y \times \{y\} \subseteq \pi_2\pi_1(V)$ . By compactness we can assume that  $A_y$  is finite and by IV.5.6,  $(1, y) = (\bigvee A_y, y) \in \pi_2\pi_1(V)$  and hence

$$(1, b) = (1, \bigvee B) \in \pi_2\pi_2\pi_1(V) = \pi_2\pi_1(V) = \nu_{\alpha-1}(U),$$

a contradiction. □