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Gutiérrez García, Javier; Kubiak, Tomasz; Picado, Jorge
Pointfree forms of Dowker’s and Michael’s insertion theorems. (English)
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The authors prove two strict insertion theorems for frame homomorphisms. When applied to the frame of all open subsets of a topological space, they are equivalent to the following insertion theorems of Dowker and Michael regarding, respectively, normal countably paracompact spaces and perfectly normal spaces.

(Dowker) A topological space $X$ is normal and countably paracompact if and only if, given $h, g : X \to \mathbb{R}$ such that $h < g, h$ is upper semicontinuous and $g$ is lower semicontinuous, there is a continuous $f : X \to \mathbb{R}$ such that $h < f < g$.

(Michael) A topological space $X$ is perfectly normal if and only if, given $h, g : X \to \mathbb{R}$ such that $h \leq g, h$ is upper semicontinuous and $g$ is lower semicontinuous, there is a continuous $f : X \to \mathbb{R}$ such that $h \leq f \leq g$ and $h(x) < f(x) < g(x)$ whenever $h(x) < g(x)$.

The authors state that, however, in the pointfree context they have been unable to provide pointfree assertions corresponding exactly to these two classical insertion results. Furthermore, they present a study of the concept of perfect normality for frames.

Hans Peter Künzi (Rondebosch)

Keywords : perfectly normal; insertion theorem; pointfree topology; frame; semicontinuous; normal; countably paracompact

Classification :

*06D22 Frames etc.
26A15 Continuity and related questions (one real variable)
54C30 Real-valued functions on topological spaces
54D15 Higher separation axioms