Gutiérrez García, Javier; Mozo Carollo, Imanol; Picado, Jorge
Normal semicontinuity and the Dedekind completion of pointfree function rings. (English) Zbl 06590289 Algebra Univers. 75, No. 3, 301-330 (2016).

This paper about the Dedekind completion of the \( \ell \)-ring \( C(L) \) of real functions on a frame \( L \). In an earlier paper [Forum Math. 27, No. 5, 2551–2585 (2015; Zbl 1332.06028)], the authors constructed the Dedekind completion of \( C(L) \) (and its bounded part \( C^*(L) \)) in terms of what they called partial continuous real functions on \( L \). The present paper supplements this earlier one by presenting three different alternative views of the completion. The first is the point-free extension of R. P. Dilworth’s [Trans. Am. Math. Soc. 68, 427–438 (1950; Zbl 0037.20205)] construction of the Dedekind completion of the \( \ell \)-ring (Dilworth viewed it as a lattice) \( C(X) \) of real-valued continuous functions on a topological space \( X \). The second view exhibits the Dedekind completion of \( C^*(L) \) for \( L \) a completely regular frame, as a function ring. More precisely, the authors show that the Dedekind completion of \( C^*(L) \) is \( C^*(\mathcal{BL}) \), where \( \mathcal{BL} \) denotes the Booleanization of \( L \). Indeed this is a function ring because, as in the classical case, every \( C \) is (isomorphic to) a \( C^* \). After drawing the attention of the reader to the fact that, in general, the Dedekind completion of an arbitrary completely regular frame cannot be a function ring, they identify a class for which it always can. The class in question is that of weakly continuously bounded frames. They show that for such a frame \( L \), the Dedekind completion of \( C(L) \) is \( C(\mathcal{GL}) \), where \( \mathcal{GL} \) is the Gleason envelope of \( L \). The latter can be realized as some closed quotient of the coproduct of \( L \) with the frame of ideals of the Booleanization of the Stone-Čech compactification of \( L \). The final construction of the Dedekind completion of \( C(L) \) is in terms of Hausdorff continuous partial real functions on \( L \). It is the point-free version of the approach in terms of interval-valued functions of the Dedekind completion of \( C(X) \) [R. Anguelov, Quaest. Math. 27, No. 2, 153–169 (2004; Zbl 1062.54017)]. The paper is well written, and treats this subject very thoroughly. Classical results in this area appear as corollaries of the authors’ point-free theorems. Furthermore, these point-free theorems cover a wider scope than their classical antecedents.

Reviewer: Themba Dube (Unisa)

MSC:

06D22 Frames, locales
06F25 Ordered algebraic structures
13J25 Commutative ordered rings
26A15 Continuity and related questions (one real variable)
54C30 Real-valued functions on topological spaces
54D15 Higher separation axioms

Keywords:
frame; locale; sublocale lattice; frame of (extended) reals; (extended) real function; continuous real function; function ring; Dedekind completion; cb-frame; normal semicontinuous real function; Booleanization; Gleason cover; partial reals; partial real function; Hausdorff continuous real function

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References:


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