Describing asymmetric frame uniformities with (pairs of) covers

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PORTUGAL

— joint work with Aleš Pultr (ITI, Charles University, Prague)

STRONG INCLUSIONS ON BIFRAMES

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$L_{\lhd}$ is a subframe of L.

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(SB6) $(L, L_{\lhd_1}, L_{\lhd_2})$ is a biframe.

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