ANÁLISE MATEMÁTICA II

Eng$^{\text{a}}$s Biomédica, Física, Geográfica, Geológica e Minas; Lic$^{\text{a}}$ Física

ANO LECTIVO DE 2004/2005 FOLHA 1

Integrais impróprios

1. Determine a natureza dos seguintes integrais impróprios e indique os seus valores no caso de convergência:

(a) $\displaystyle\int_1^{+\infty} \frac{2}{x^2}\,dx$; (b) $\displaystyle\int_{-\infty}^{0} e^x\,dx$; (c) $\displaystyle\int_{-\infty}^{+\infty} x^2\,dx$;

(d) $\displaystyle\int_4^{+\infty} \frac{1}{x}\,dx$; (e) $\displaystyle\int_{-\infty}^{0} x\,5^{-x^2}\,dx$; (f) $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{x^4+9}\,dx$;

(g) $\displaystyle\int_{-1}^{1} \frac{1}{x^2}\,dx$; (h) $\displaystyle\int_{-1}^{1} \frac{3}{x}\,dx$; (i) $\displaystyle\int_0^4 \frac{1}{(x-3)^2}\,dx$;

(j) $\displaystyle\int_{\frac{1}{2}}^{2} \frac{1}{x(\ln x)^{\frac{1}{5}}}\,dx$; (k) $\displaystyle\int_{-1}^{1} \frac{\operatorname{ch}^3 x}{\operatorname{sh} x}\,dx$; (l) $\displaystyle\int_{3/2}^{2} \frac{1}{\sqrt{x-1}-\sqrt[3]{x-1}}\,dx$;

(m) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{1+x^2}\,dx$; (n) $\displaystyle\int_{-\infty}^{+\infty} \sin x\,dx$; (o) $\displaystyle\int_1^{+\infty} x^2 e^{-x}\,dx$;

(p) $\displaystyle\int_{-1}^{+\infty} \frac{1}{x(x-2)}\,dx$; (q) $\displaystyle\int_{\frac{\pi}{2}}^{+\infty} e^{-2x}\cos x\,dx$; (r) $\displaystyle\int_0^1 \frac{\operatorname{arc\,sin} x}{x^2}\,dx$.

2. Mostre que

(a) $\displaystyle\lim_{t\to+\infty}\int_{-t}^{t} \frac{x}{1+x^2}\,dx$ existe;

(b) $\displaystyle\int_{-\infty}^{+\infty} \frac{x}{1+x^2}\,dx$ é divergente.

3. Determine a natureza dos seguintes integrais impróprios:

(a) $\displaystyle\int_0^{1/2} \frac{1}{(1-x)\sqrt{x}}\,dx$;

(b) $\displaystyle\int_0^{+\infty} \frac{1}{e^x+e^{-x}}\,dx$;

(c) $\displaystyle\int_0^{3/2} \frac{1}{\sqrt{9-4x^2}}\,dx$;

(d) $\displaystyle\int_0^1 \frac{1}{x\sqrt{1-x^2}}\,dx$.

4. Determine a natureza dos seguintes integrais impróprios, usando os critérios de comparação:

(a) $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{x^2+6x+12}\,dx$; (b) $\displaystyle\int_{0}^{+\infty} \frac{x}{x^3+x^2+1}\,dx$;

(c) $\displaystyle\int_{0}^{+\infty} \sin x\,dx$; (d) $\displaystyle\int_{1}^{+\infty} \frac{\operatorname{sh}(x+1)}{x}\,dx$;

(e) $\displaystyle\int_{1}^{+\infty} \frac{1}{2x+\sqrt[3]{x^2+1}+5}\,dx$; (f) $\displaystyle\int_{0}^{100} \frac{1}{\sqrt[3]{x}+2\sqrt[4]{x}+x^3}\,dx$;

(g) $\displaystyle\int_{1/2}^{2} \frac{1}{x\ln^{1/5} x}\,dx$; (h) $\displaystyle\int_{\pi/2}^{+\infty} \frac{\sin x}{x^2}\,dx$;

(i) $\displaystyle\int_{0}^{1} \frac{1}{\ln(1+\sqrt{x})}\,dx$; (j) $\displaystyle\int_{1}^{+\infty} \frac{\ln x}{1+x^3}\,dx$;

(k) $\displaystyle\int_{-1}^{1} t^{-1}e^{-t}\,dx$; (l) $\displaystyle\int_{-1}^{1} t^{-1/3}e^{-t}\,dx$;

(m) $\displaystyle\int_{0}^{4} \frac{x}{x^2-1}\,dx$; (n) $\displaystyle\int_{-\infty}^{\pi/2} \frac{\sin x}{x^2}\,dx$.

5. (a) Mostre que $\ln(1+x) < x$ para todo $x \in]0,+\infty[$.

   (b) Utilizando alínea anterior, determine, justificando convenientemente a sua resposta, a natureza do integral

$$\int_1^{\infty} \ln\left(1+\frac{1}{x^2}\right)\,dx.$$

6. Determine a natureza dos seguintes integrais impróprios:

   (a) $\displaystyle\int_1^{+\infty} \frac{\cos x}{x^2}dx$;

   (b) $\displaystyle\int_0^1 \cos\frac{1}{x}dx$.

7. Determine a natureza do integral impróprio

$$\int_2^{+\infty} \frac{x^n}{x^3+x+1}\,dx$$

em função dos valores de n.

8. (a) Mostre, utilizando um critério de comparação, que o integral impróprio

$$\int_0^{+\infty} \frac{t}{\sqrt{t^4+1}}\,dt$$

é divergente.

   (b) Calcule

$$\lim_{x\to+\infty} \frac{1}{x^3}\int_0^{x^2} \frac{t}{\sqrt{t^4+1}}\,dt.$$