Departamento de Matemática da Universidade de Coimbra

Análise Matemática II

Eng$^{\text{a}}$s Biomédica, Física, Geográfica, Geológica e Minas; Lic$^{\text{a}}$ Física

Ano lectivo de 2004/2005 — folha 3

Sucessões Numéricas

27. Estude a convergência das sucessões a seguir definidas.

(a) $a_n = n(n-1)$; (b) $a_n = \dfrac{3+5n^2}{n+n^2}$; (c) $a_n = \dfrac{2^n}{3^{n+1}}$; (d) $a_n = 2+\cos(n\pi)$;

(e) $a_n = \operatorname{arctg}(2n)$; (f) $a_n = \dfrac{\ln(n^2)}{n}$; (g) $a_n = \dfrac{n!}{2^n}$; (h) $a_n = (-1)^n \sin(1/n)$;

(i) $n!\left(\dfrac{\pi}{2} - \operatorname{arc\,tg} n!\right)$; (j) $n \operatorname{sh}(1/n)$.

28. Usando o critério das sucessões enquadradas, calcule:

(a) $\lim \dfrac{1}{n\sqrt{n^{-4}+2n^{-2}+7}}$; (b) $\lim \left[\sin(n)n^{-1} - \cos(n)e^{-n}\right]$; (c) $\lim \dfrac{1}{n!}$.

29. Verifique se as seguintes sucessões são convergentes e indique os seus limites em caso afirmativo:

(a) $a_n = \begin{cases} \dfrac{n+1}{n} & \text{se } n \text{ par} \\[2ex] \dfrac{n}{n+1} & \text{se } n \text{ ímpar} \end{cases}$; (b) $b_n = \cos\left(\frac{2}{3}\pi n\right)$.

30. Estude a monotonia das sucessões de termo geral

(a) $a_n = \cos\left(\frac{2}{3}\pi n\right)$; (b) $a_n = \dfrac{2n-3}{3n+4}$; (c) $a_n = \begin{cases} \dfrac{1-n}{n} & \text{se } n \text{ par} \\[2ex] \dfrac{2}{n} & \text{se } n \text{ ímpar} \end{cases}$.

Séries Numéricas

31. Determine a sucessão de somas parciais e calcule, se possível, a soma das seguintes séries:

(a) $\displaystyle\sum_{n=1}^{+\infty} (-1)^{n-1}$; (b) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{2}{3}\right)^n$; (c) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}$; (d) $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{(3n-1)(3n+5)}$.

32. Usando a condição necessária de convergência, prove que as seguintes séries são divergentes:

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^n}{n^3}$; (b) $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{\sqrt{3n^2-2}}$; (c) $\displaystyle\sum_{n=1}^{+\infty} \sqrt[n]{\frac{2n}{3^n}}$.

33. Determine a natureza das seguintes séries:

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{n+1}$; (b) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log n}{n}$; (c) $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{n^2+1}$; (d) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log n}{n^2}$;

(e) $\displaystyle\sum_{n=1}^{+\infty} \frac{8^n\, n!}{n^n}$; (f) $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{n^3-1}$; (g) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{8^n} + \frac{1}{n(n+1)}$; (h) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n}}{n+4}$;

(i) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sin\left(\frac{\pi}{50}\right)}{2^n}$; (j) $\displaystyle\sum_{n=1}^{+\infty} 5\left[1-\cos\left(\frac{1}{n}\right)\right]$; (k) $\displaystyle\sum_{n=1}^{+\infty} \left(1-\frac{\pi}{n+3}\right)^{n^2}$; (l) $\displaystyle\sum_{n=1}^{+\infty} \frac{\log n}{\sqrt[4]{n^5}}$;

(m) $\displaystyle\sum_{n=1}^{+\infty} \left[n\sin\left(\frac{\pi}{3n}\right)\right]^n$; (n) $\displaystyle\sum_{n=1}^{+\infty} \frac{n!}{2^{n+1}}$.

34. Verifique se as seguintes séries alternadas são absolutamente ou simplesmente convergentes:

(a) $\displaystyle\sum_{n=2}^{+\infty} (-1)^n \frac{1}{\log n}$; (b) $\displaystyle\sum_{n=2}^{+\infty} (-1)^n \frac{n}{n^2-2}$; (c) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \frac{e^n \log n}{(n+1)!}$;

(d) $\displaystyle\sum_{n=2}^{+\infty} (-1)^{n+1} \operatorname{arctg}\left(\frac{n}{e^n}\right)$; (e) $\displaystyle\sum_{n=2}^{+\infty} (-1)^{n+1} \log\left(\frac{n+1}{n}\right)$; (f) $\displaystyle\sum_{n=2}^{+\infty} (-1)^n \frac{\log n}{n}$.

35. Determine a natureza das seguintes séries:

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n}{n2^n}$; (b) $\displaystyle\sum_{n=1}^{+\infty} (-1)^{n+1}\left(\frac{6}{5}\right)^n$; (c) $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$;

(d) $\displaystyle\sum_{n=1}^{+\infty} \frac{\cos n}{n^2}$; (e) $\displaystyle\sum_{n=1}^{+\infty} \left(\frac{(-1)^n}{n+3} - \frac{1}{n^2+3}\right)$; (f) $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n^3+1}}{3n^3+4n^2+2}$;

(g) $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$; (h) $\displaystyle\sum_{n=1}^{+\infty} \frac{2^3.\,4^3\ldots(2n)^3}{1^3\,3^3\ldots(2n-1)^3}$.

36. Calcule os limites das sucessões de termos gerais:

(a) $\displaystyle\frac{4^{n-1}}{(4n-1)!}$; (b) $\displaystyle\frac{(n!)^2}{(2n)!}$; (c) $\displaystyle\left(\frac{1}{3^n+1}\right)^n$.

37. Determine quais os valores de k para os quais cada uma das seguintes séries é convergente ou divergente:

(a) $\displaystyle\sum_{n=1}^{+\infty} \frac{(n-1)!\,3^n}{n^{kn}}$; (b) $\displaystyle\sum_{n=1}^{+\infty} \left(1+\frac{1}{n}\right)^n k^n$.