Ergodic theory for operators and applications to generalized Cesàro operators

José Bonet*

Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, Spain

jbonet@mat.upv.es

In the first part of this lecture we will review classical results about power bounded and mean ergodic operators acting on Banach and more general spaces. Theorems by Eberlein, Yosida and Lin will be stated. Compactness plays an important role in these results. Some new abstract results will be presented. They will be utilized to investigated the behaviour of generalized Cesàro operators when acting on sequence spaces and spaces of analytic functions on the disc of the complex plane.

The generalized Cesàro operators C_t, for $t \in [0,1]$, acts from $\mathbb{C}^{\mathbb{N}_0}$ into itself (with $\mathbb{N}_0 := 0, 1, 2, \ldots$) is given by

$$C_t x := \left(\frac{t^n x_0 + t^{n-1} x_1 + \ldots + x_n}{n+1} \right)_{n \in \mathbb{N}_0}, \quad x = (x_n)_{n \in \mathbb{N}_0} \in \mathbb{C}^{\mathbb{N}_0}. \tag{1}$$

For $t = 0$ note that C_0 is a diagonal operator and for $t = 1$ that C_1 is the classical Cesàro averaging operator. These operators act continuously in many classical Banach sequence spaces such as ℓ^p, c_0, c. In the setting of analytic functions C_t has the integral representation $C_t f(0) := f(0)$ and

$$C_t f(z) := \frac{1}{z} \int_0^z \frac{f(\zeta)}{1 - t\zeta} \, d\zeta, \ z \in \mathbb{D} \setminus \{0\}, \tag{2}$$

for every $f \in H(\mathbb{D})$.

References

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[2] A. A. Albanese, J. Bonet, W. J. Ricker, Generalized Cesàro operators in weighted Banach spaces of analytic functions with sup-norms. Collectanea Math. (to appear).

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