

Why didn't locale-theorists discover DeMorganization?

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As is well known, the logical principle called 'De Morgan's Law' is the analogue for toposes of the topological property of extremal disconnectedness. In 2008 my then student Olivia Caramello showed that every topos has a largest dense subtopos satisfying De Morgan's law; this immediately implies that every locale has a largest dense sublocale which is extremally disconnected, but at the time we had no purely locale-theoretic proof of that fact. In this talk I shall present a simple frame-theoretic proof of the existence of 'DeMorganization', based on a technique which I introduced in 1989 when studying fibrewise closed sublocales, and try to explain why it was not discovered earlier. I shall also present a first contribution to the study of locales which are 'DM-averse' in the sense that their DeMorganization coincides with their Booleanization, by showing that all metric spaces are DM-averse. And I shall discuss possible connections with recent work of Rick Ball and Joanne Walters-Wayland on smallest flat and C^* -embedded sublocales.