

# Spectra and subspectra arising from $\ell$ -groups and commutative rings

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The spectrum of an Abelian  $\ell$ -group is defined as the set of all its prime  $\ell$ -ideals endowed with the hull-kernel topology. The real spectrum of a commutative unital ring is an ordered analogue of its Zariski spectrum. We give a complete list of the containments and non-containments between the classes of  $\ell$ -spectra and real spectra, and their spectral subspaces, for  $\ell$ -groups and rings, highlighting the differences between the cases of structures of cardinality either countable,  $\aleph_1$ , or  $\aleph_2$ . We also give a hint of the methods used: semilinear algebra / real algebraic geometry (cases  $\aleph_0, \aleph_1$ ), category theory, infinite combinatorics, and logic (beyond  $\aleph_1$ , especially  $\aleph_2$ ).