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Generalized Eilenberg Theorem

Eilenberg proved that varieties of finite monoids bijectively correspond to varieties of regular languages, i.e., classes of regular languages closed under the boolean set-theoretical operations, derivations, and preimages under monoid homomorphisms. We prove a much more general result, based on combining coalgebraic and algebraic methods.

We work with a locally finite variety C of algebras (instead of just boolean algebras). Then we form the predual category D which means that D is the ind-completion of the dual of all finitely presentable objects of C. The role of finite monoids is now taken by finite bimonoids in D. Example: if C are distributive lattices, then D are posets. We thus prove the result of [2] that varieties of finite ordered monoids bijectively correspond to lattice-varieties of regular languages (closed under union and intersection but not necessarily under complement). Another example: if C are vector spaces over the binary field, then D equals C, and the role of finite monoids is taken over by algebras over the field (in the classical sense of K-algebras). We thus prove that varieties of finite K-algebras bijectively correspond to vector-varieties of regular languages (closed, instead of under boolean operations, under symmetric difference).

References:

[1] Eilenberg, S., *Automata, languages and machines*, vol. B., Academic Press [Harcourt Brace Janovich Publishers], New York (1976).

[2] Gehrke, M., Griegorieff, S., Pin, J.É., Duality and equational theory of regular languages, Proc. ICALP 2010, Part II. *Lecture Notes Comput. Sci.*, Springer 5126 (2008) 246–257.

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