LINEAR STRUCTURES ON LOCALES

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 Construction of certain vector bundles (Fell bundles) on topological groupoids from C*-algebras equipped with "diagonals" [Renault, Kumjian, Exel, Buss]

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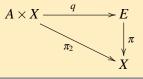
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- Joint work with João Paulo Santos (IST):
 - Open quotients of trivial vector bundles; arXiv:1510.06329
 - Linear structures on locales, Theory Appl. Categ. 31 (2016) 502-541

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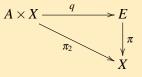
DEFINITION

By a **quotient vector bundle** is meant a triple (π, A, q) consisting of a "linear bundle" $\pi: E \to X$ on a topological space X, a topological vector space (TVS) A, and a continuous open surjection $q: A \times X \to E$ that makes the following commute:



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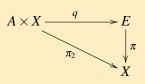
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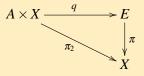


EXAMPLE

Any Banach bundle on a locally compact Hausdorff space, with $A = C_0(E)$ and q = eval: for each section $s: X \to E$ we have q(s, x) = s(x).

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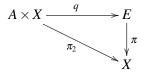
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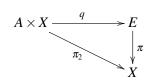
Any Fell bundle on an étale locally compact Hausdorff groupoid, with $A = C_r^*(E)$ and q = eval.



Notation: $E_x := \pi^{-1}(\{x\})$ is the **fiber** over x

Some properties:

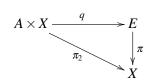
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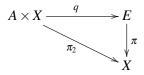


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- For each $x \in X$, the map $q_x : A \to E_x$ defined by $q_x(a) = q(a,x)$ is a continuous open surjection, so E_x is a quotient TVS of A.
- The subspace topology of $E_x \subset E$ coincides with the quotient topology.

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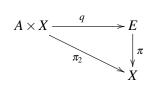


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- Solution The subspace topology of E_x ⊂ E coincides with the quotient topology.
- **(**) Each $a \in A$ defines a section $\hat{a}: X \to E$ by $\hat{a}(x) = q(a, x)$.

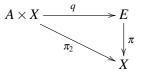
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- **(**) Each $a \in A$ defines a section $\hat{a}: X \to E$ by $\hat{a}(x) = q(a, x)$.
- π "has enough sections": for all $e \in E_x$ there is $a \in A$ such that $\hat{a}(x) = e$.



 $SubA := \{linear subspaces of A\}$ $MaxA := \{closed linear subspaces of A\}$

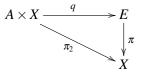
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The **kernel map** $\kappa : X \to \text{Sub}A$ is defined by

$$\kappa(x) = \{a \in A \mid q(a, x) = 0\}$$

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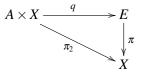
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$$\bullet E_x \cong A/\kappa(x).$$

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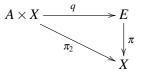
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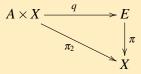
- $\bullet E_x \cong A/\kappa(x).$
- **2** The fibers E_x are Hausdorff iff $\kappa : X \to MaxA$.
- κ determines E and q uniquely up to isomorphisms.

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THEOREM

Let A be a TVS, X a topological space, and $\kappa : X \to SubA$ any map. Obtain a commutative diagram



by constructing *E* as the quotient of $A \times X$ defined by

$$(a,x) \sim (b,y) \iff x = y \text{ and } a - b \in \kappa(x)$$
.

The quotient map $q: A \times X \to E$ is open iff κ is continuous with respect the lower Vietoris topology of SubA.

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DEFINITION

SubA is the **classifying space** for QVBs with TVS A. The QVB

$$UA := (\pi_A : UA \to SubA, A, q_A)$$

classified by the identity on SubA is the **universal QVB for** A.

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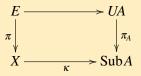
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COROLLARY

Any $QVB(\pi, A, q)$ is the pullback of UA along the kernel map:

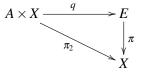


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- Closed balls topology on MaxA (A normed) classifies QVBs with continuous norm.
- With A a Banach space and X Hausdorff, MaxA classifies QVBs which are Banach bundles.

Let $\mathscr{A} = (\pi : E \to X, A, q)$ be a QVB:



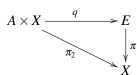
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DQC

Let $\mathscr{A} = (\pi : E \to X, A, q)$ be a QVB:



Notation: supp[°] $\hat{a} = int\{x \in X \mid q(a,x) \neq 0\}$ is the **open support of** \hat{a} .

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$$\mathscr{A} = (\pi : E \to X, A, q)$$
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 $A \times X \xrightarrow{q} E$

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 π

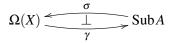
$$\Omega(X) \xrightarrow[a \in V]{} SubA$$

$$\sigma(V) = \bigcup_{a \in V} supp^{\circ} \hat{a}$$

$$\gamma(U) = \{a \in A \mid supp^{\circ} \hat{a} \subset U\}$$

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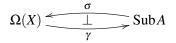
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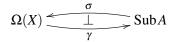
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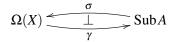
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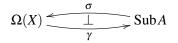


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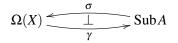
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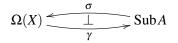
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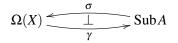
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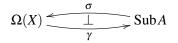
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If X is sober and the zero section is closed in E then the QVB is spectral.



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E.g., Banach bundles and Fell bundles as before.

THEOREM

Let A be a locally convex space. Then MaxA (with the lower Vietoris topology) is sober.

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If A is locally convex the universal QVB (π, A, q) with Hausdorff fibers is not spectral, even though MaxA is sober.

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DEFINITION

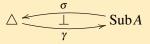
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DEFINITION

Let \triangle be a locale, and A a TVS, equipped with an adjunction



such that

$$\mathfrak{k} := \gamma|_{\Sigma(\triangle)} : \Sigma(\triangle) \to \mathrm{Sub}A$$

is continuous.

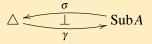
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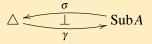
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The triple (A, σ, γ) is called a **linear structure** on \triangle , and $\mathfrak{A} = (\triangle, A, \sigma, \gamma)$ is a **linearized locale**.

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EXAMPLE

Every spectral vector bundle \mathscr{A} yields a linearized locale $\Omega(\mathscr{A}) = (\triangle, A, \sigma, \gamma)$, with $\triangle = \Omega(X)$, as described earlier.

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Conversely, every linearized locale $\mathfrak{A} = (\triangle, A, \sigma, \gamma)$ defines a QVB $\Sigma(\mathfrak{A}) = (\pi : E \to X, A, q)$ with $X = \Sigma(\triangle)$ and kernel map $\mathfrak{k} = \gamma|_X$.

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COROLLARY

Correspondence

Spectral vector bundles
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 Linearized locales

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• Let $\mathscr{A} = (\pi : E \to X, A, q)$ and $\mathscr{B} = (\rho : F \to Y, B, r)$ be QVBs.

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- Let $\mathscr{A} = (\pi : E \to X, A, q)$ and $\mathscr{B} = (\rho : F \to Y, B, r)$ be QVBs.
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If the above is an equivalence we say that f is **strict**.

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- Let $\mathscr{A} = (\pi : E \to X, A, q)$ and $\mathscr{B} = (\rho : F \to Y, B, r)$ be QVBs.
- A morphism $f: \mathscr{B} → \mathscr{A}$ should "induce" a continuous linear map
 $f^*: A → B$.

DEFINITION

A morphism $f: \mathscr{B} \to \mathscr{A}$ is a pair (f_{\flat}, f^*) such that

- $f_{\flat}: Y \to X$ is continuous;
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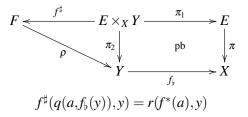
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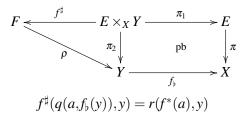
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Obtain category $QVBun_{\Sigma}$ of spectral vector bundles.

• There is a uniquely defined continuous fiberwise linear map f^{\sharp} that makes the following diagram commute:

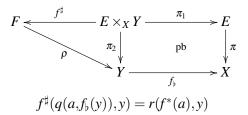


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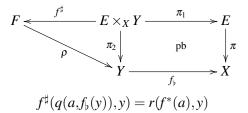
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- Different from a morphism of fiber bundles $\begin{array}{c} \rho \downarrow & \stackrel{f_1}{\to} E\\ \rho \downarrow & \downarrow \pi \end{array}$

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DEFINITION

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DEFINITION

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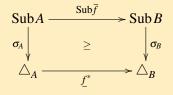
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satisfying, for all $V \in \text{Sub}A$, the inclusion $\sigma_B(\overline{f}(V)) \subset f^*(\sigma_A(V))$:



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DEFINITION

(cont.)

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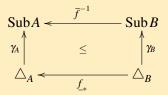
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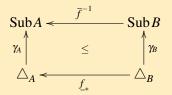
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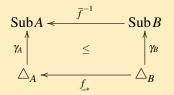


This defines the category of linearized locales LinLoc.

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This defines the category of linearized locales LinLoc.

If the above commutation relations are strict the morphism $\mathfrak f$ is called strict.

THEOREM

The correspondence between spectral vector bundles and linearized locales extends to an adjunction

(and to a restricted adjunction considering only strict morphisms).