On nonparametric estimation of boundary measures

Antonio Cuevas, Departamento de Matemáticas, Universidad Autónoma de Madrid, Spain
antonio.cuevas@uam.es

Abstract: The measure of the boundary ∂G of a compact body $G \subset [0,1]^d$ can be expressed in terms of the Minkowski content defined by

$$L_0(G) = \lim_{\epsilon \to 0} \frac{\mu\left(B(\partial G, \epsilon)\right)}{2\epsilon}.$$

Here μ denotes the Lebesgue measure on $\mathbb{R}^d$ and $B(G,\epsilon)$ stands for the parallel set $B(G,\epsilon) = \cup_{x \in G} B(x,\epsilon)$, where $B(x,\epsilon)$ is the closed ball with center $x \in \mathbb{R}^d$ and radius $\epsilon > 0$.

This concept is less general than the $(d-1)$-dimensional Hausdorff measure of ∂G (which coincides with $L_0(G)$ in regular cases) but it is more suitable for the estimation of the boundary measure with statistical methods.

To be more concrete, our methods can be used in those cases where the sampling information consist of random observations in $[0,1]^d$ in such a way that for each observation we are able to decide whether or not it belongs to G.

In these situations a natural nonparametric estimator of $L_0(G)$ can be defined. We will present some results concerning consistency, convergence rates and asymptotic normality of such estimator.

The practical aspects of these ideas in image analysis will be also briefly commented.

Most of this talk is a summary of recent joint work with Inés Armendáriz, Ricardo Fraiman (both from Universidad de San Andrés, Buenos Aires, Argentina) and Alberto Rodríguez-Casal (Universidad de Santiago de Compostela, Spain).

References

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