2-dimensional monadicity

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It has been known since [1] that the varieties of 2-dimensional universal algebra exhibit behaviour which cannot be captured using 2-category theory alone, but require the ability to speak of the subcollection of strict morphisms. A simple example is that the 2-category of monoidal categories and lax monoidal functors has products, formed as in Cat, with moreover the product projections "strict monoidal" and jointly detecting "strictness". In seeking to understand more complex phenomena of a similar flavour the authors of [2] were led to introduce the notion of an F-category, which is a 2-category with a specified subcollection of "tight" morphisms. For example there is an F-category of monoidal categories and lax monoidal functors, with tight morphisms the strict ones, and now the above property can be described as a limit lifting property of the forgetful F-functor from there to Cat.

In the present talk I will describe a series of properties of such forgetful F-functors to 2-categories, whereby any F-functor satisfying them must be the forgetful F-functor from an F-category of algebras for a 2-monad, souping up Beck's theorem from the strict world to cover each weaker kind of morphism.

References

[1] R. Blackwell, G.M. Kelly and A.J. Power, *Two-Dimensional Monad Theory*, J. Pure Appl. Algebra 59 (1989), 1-41.

[2] S. Lack and M. Shulman, *Enhanced 2-categories and limits for lax morphisms*, Advances in Mathematics (2011), 294-256.