

Monads in \mathbf{Top}_0 from bicompletion of functorial quasi-uniformities

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The forgetful functor $T_u : \mathbf{Unif}_0 \rightarrow \mathbf{Tych}$ from the category of Hausdorff uniform spaces to that of the completely regular Hausdorff spaces serves as introduction. Let the pointed endofunctor (K, k) denote the classic Cauchy completion in \mathbf{Unif}_0 . Every section F of T_u induces a pointed endofunctor $(R^F, r^F) := (T_u K F, T_u k F)$ in \mathbf{Tych} . The coarsest T_u -section thereby gives the epireflection in \mathbf{Tych} to the compact Hausdorff spaces; the finest gives the epireflection to the topologically complete spaces; and every epireflective subcategory of \mathbf{Tych} between these two extremes is induced under the above procedure by at least one T_u -section F .

Instead of \mathbf{Tych} we really want to study the T_0 -topological spaces, \mathbf{Top}_0 . Because the latter carry order-theoretic information, one drops the symmetry axiom of \mathbf{Unif}_0 , thus extending it to \mathbf{QU}_0 , the T_0 quasi-uniform spaces. Now we have two forgetful functors: $T : \mathbf{QU}_0 \rightarrow \mathbf{Top}_0$ (take the first topology) and $T_b : \mathbf{QU}_0 \rightarrow \mathbf{2Tych}$ (take both topologies, getting the "pairwise" or "bi" Tychonoff bitopological spaces). The T_b -sections and their associated structures play a role remarkably similar to that of the T_u -sections, but sharply different from that of the T -sections. The reason is that T_u and T_b preserve epimorphisms, while T does not. Thus, if the induced pointed endofunctors in \mathbf{Tych} or $\mathbf{2Tych}$ admit monad multiplications they will be reflections. In \mathbf{Top}_0 the existence of the monad multiplication is of interest (see [1], [2]) and can be characterized by extending the pointed endofunctors from \mathbf{Top}_0 to $\mathbf{2Tych}$.

REFERENCES

- [1] G.C.L. Brümmer and H.-P.A. Künzi, *Idempotency of extensions via the bicompletion*, Appl. Categ. Struct. 15 (2007) 499–509.
- [2] G.C.L. Brümmer, H.-P.A. Künzi and M. Sioen, *More on upper bicompletion-true functorial quasi-uniformities*, Topology Appl. 158 (2011) 1937–1941.