On simple Kock-Zoberlein monads

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In their seminal paper [2], Cassidy, Hébert and Kelly built a factorization system for each simple reflective subcategory of a category with pullbacks, constructed via a pullback lifting of the reflexion.

In this talk we will work on a partially-ordered enriched category $\mathcal{C}$ and show that, when $\mathcal{C}$ has comma objects, an analogous construction can be made when a Kock-Zoberlein (or lax idempotent) monad is given. By analogy, we introduce simple Kock-Zoberlein monads, and show that they induce a lax orthogonal factorization system. As examples of simple Kock-Zoberlein monads we have filter monads in topological T_0-spaces (as studied in [7] and [1]) and relative presheaf monads in enriched separated V-categories – or, more generally, in separated (T, V)-categories – for a quantale V and a monad T satisfying (BC) (as studied in [6] and [3]).

Since simple Kock-Zoberlein monads have Kock-Zoberlein liftings to the comma categories $\mathcal{C} \downarrow B$, for any object B of $\mathcal{C}$, Escardo's results on injectivity (see [4] and [5]) can be used to characterize injectivity of morphisms with respect to special classes of embeddings (as done in [1]).

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