

# Semidirect products and Split Short Five Lemma in normal categories

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The categorical definition of semidirect products was introduced by D. Bourn and G. Janelidze in [2], where they proved that, in the category of groups, this notion coincides with the classical one.

A characterization of pointed categories with categorical semidirect products was given in [3]. The existence of such products imply, in particular, that the category is protomodular, i.e. the Split Short Five Lemma holds.

We will extend the notion of semidirect product to non-protomodular categories such that internal actions, as defined in [1], are induced by points. This fact is true, for example, in any pointed variety, and also in any Barr-exact Mal'cev normal category, as showed in [3]. Under this condition, we will show that internal actions are equivalent to regular points (i.e. points such that the kernel and the section are jointly strongly epimorphic) if and only if a weaker version of the Split Short Five Lemma, only for regular points, holds. Moreover, we will show that the weaker Split Short Five Lemma holds when the category is normal.

## REFERENCES

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- [3] N. Martins-Ferreira, M. Sobral, *On categories with semi-direct products*, J. Pure and Appl. Algebra 216 (2012), 1968–1975.

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