

# Notes on exact meets and joins

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Exact meets and joins appeared in various contexts, for instance in R. N. Ball's study of essential extensions of lattices. A transparent (albeit a bit misleading) illustration of the phenomenon is the exact meet in the lattices  $\Omega(X)$  of open sets in  $T_D$ -spaces  $X$ : here this is precisely the case of the intersections of open sets that happen to be open (in other words, the meets in  $\Omega(X)$  that are set intersections).

Exact meets and joins will be discussed, first, in the general context of lattices (distributivity or any sort of infinite completeness not assumed). Then we will discuss the exact meets in frames (where all the joins are exact), in spatial frames, and finally in Scott topologies.

Exactness is connected on the one hand with distributivity, on the other hand (in frames) with the behaviour of meets and joins of sublocales. The example of open intersections mentioned above is specific for the  $T_D$ -spatial case. In the more general cases, the exactness is associated, rather, with closed unions of closed subobjects; the "open intersection property" is stronger and it turns out to be also of interest of its own.

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