The cuboid lemma and Mal'tsev categories

Diana Rodelo [*]

We prove that a regular category $\mathbb{C}$ is a Mal'tsev category if and only if a strong form of the denormalised 3-by-3 Lemma [1] holds true in $\mathbb{C}$. In this version of the 3-by-3 Lemma, the vertical exact forks are replaced by pullbacks of split epimorphisms along arbitrary morphism. The shape of the diagram it determines suggests to call it the Cuboid Lemma. This new characterisation of regular categories that are Mal'tsev categories (= 2-permutable) is similar to the one obtained in [2, 3] for Goursat categories (= 3-permutable). We also analyse the restricted "pointed" counterpart of the Cuboid Lemma, as well as the extended "relative" context [4].

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