Lattices of Subobjects and Quotients in Locally Finitely Presentable Categories

Hans-E. Porst

Department of Mathematics

Universität Bremen

Coimbra, July 12, 2012

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Outline

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Motivation

Whatever can be said about varieties can be proved categorically!

What about

1. The subobjects of every algebra in a variety form an algebraic lattice.
2. The quotients (equivalently: kernel pairs = equivalence relations) of every algebra in a variety form an algebraic lattice.

Are there categorical proofs?

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Motivation

Whatever can be said about varieties can be proved categorically!

What about

1. The subobjects of every algebra in a variety form an algebraic lattice.
2. The quotients (equivalently: kernel pairs = equivalence relations) of every algebra in a variety form an algebraic lattice.

Are there categorical proofs?

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Stated more categorically

Since a poset P is an algebraic lattice iff P, considered as a category, is locally finitely presentable, we might rephrase the above as follows:

For certain locally finitely presentable categories $\mathcal{A}$ (the varieties) it holds that for every $\mathcal{A}$-object A

1. The subobjects of A form a locally finitely presentable category.
2. The quotients of A form a locally finitely presentable category.

Maybe, this even holds for *all* locally finitely presentable (l.f.p.) categories?

Stated more categorically

Since a poset *P* is an algebraic lattice iff *P*, considered as a category, is locally finitely presentable, we might rephrase the above as follows:

For certain locally finitely presentable categories $\mathcal{A}$ (the varieties) it holds that for every $\mathcal{A}$-object *A*

1. The subobjects of *A* form a locally finitely presentable category.
2. The quotients of *A* form a locally finitely presentable category.

Maybe, this even holds for *all* locally finitely presentable (l.f.p.) categories?

Stated more categorically

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Since a poset P is an algebraic lattice iff P, considered as a category, is locally finitely presentable, we might rephrase the above as follows:

For certain locally finitely presentable categories $\mathcal{A}$ (the varieties) it holds that for every $\mathcal{A}$-object A

1. The subobjects of A form a locally finitely presentable category.
2. The quotients of A form a locally finitely presentable category.

Maybe, this even holds for *all* locally finitely presentable (l.f.p.) categories?

Results by closure properties

Our results will mainly be consequences of the following familiar closure properties of l.f.p. categories

Theorem (Slice Category Theorem)

For every l.f.p. category $\mathcal{A}$ and every $A \in \mathcal{A}$ the categories $A \downarrow \mathcal{A}$ and $\mathcal{A} \downarrow A$ are l.f.p. categories.

Theorem (Subcategory Theorem)

Every full subcategory of l.f.p. category is l.f.p., provided that it is closed under limits and directed colimits.

What kind of subobjects?

Motivation
Subobject lattices
Quotient lattices
The lattice of internal equivalence relations
Examples

While, in a variety, *subobject* clearly means “subobject presented by a monomorphism”, in an arbitray l.f.p. category there are the following natural options

1. subobject presented by a *monomorphism*,
2. subobject presented by a *strong (extremal) monomorphism*,
3. subobject presented by a *regular monomorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi,strong mono) factorization structure, and that (as e.g. in **Poset**) the natural subobjects are those presented by strong (= regular) monomorphisms. Moreover, even in varieties, strong and regular monomorphisms need not coincide (see e.g. **Sgr**).

What kind of subobjects?

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

While, in a variety, *subobject* clearly means “subobject presented by a monomorphism”, in an arbitray l.f.p. category there are the following natural options

1. subobject presented by a *monomorphism*,
2. subobject presented by a *strong (extremal) monomorphism*,
3. subobject presented by a *regular monomorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi,strong mono) factorization structure, and that (as e.g. in **Poset**) the natural subobjects are those presented by strong (= regular) monomorphisms.
Moreover, even in varieties, strong and regular monomorphisms need not coincide (see e.g. **Sgr**).

What kind of subobjects?

While, in a variety, *subobject* clearly means "subobject presented by a monomorphism", in an arbitray l.f.p. category there are the following natural options

1. subobject presented by a *monomorphism*,
2. subobject presented by a *strong (extremal) monomorphism*,
3. subobject presented by a *regular monomorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi,strong mono) factorization structure, and that (as e.g. in **Poset**) the natural subobjects are those presented by strong (= regular) monomorphisms. Moreover, even in varieties, strong and regular monomorphisms need not coincide (see e.g. **Sgr**).

Notation

For an $\mathcal{A}$-object A we denote by

1. $Sub_m(A)$ the full subcategory of $\mathcal{A} \downarrow A$ spanned by all monomorphims $B \xrightarrow{m} A$,
2. $Sub_{str}(A)$ the full subcategory of $\mathcal{A} \downarrow A$ spanned by all strong monomorphims $B \xrightarrow{m} A$,
3. $Sub_{reg}(A)$ the full subcategory of $\mathcal{A} \downarrow A$ spanned by all regular monomorphims $B \xrightarrow{m} A$.

Clearly, then the poset (l.f.p. categories are wellpowered) of *all* (all *strong*, all *regular*) subobjects of A is (isomorphic to) a skeleton of $Sub_m(A)$, $Sub_{str}(A)$ and $Sub_{reg}(A)$, respectively.

Questions

We therefore are to answer the following questions for an object A in a l.f.p. category $\mathcal{A}$

1. Is the category $Sub_m(A)$ locally finitely presentable?
2. Is the category $Sub_{str}(A)$ locally finitely presentable?
3. Is the category $Sub_{reg}(A)$ locally finitely presentable?

In any l.f.p. category, each of the classes

Mono	of *all* monomorphisms,
$Mono_{str}$	of *strong* monomorphisms,
$Mono_{reg}$	of *regular* monomorphisms

is closed under intersections. Thus, the categories $Sub_m(A)$, $Sub_{str}(A)$ and $Sub_{reg}(A)$ are closed under products in $\mathcal{A} \downarrow A$.

Hence, by the Slice and the Subcategory Theorem, they are locally finitely presentable, provided that they are closed under directed colimits, too.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

In any l.f.p. category, each of the classes

Mono	of *all* monomorphisms,
$Mono_{str}$	of *strong* monomorphisms,
$Mono_{reg}$	of *regular* monomorphisms

is closed under intersections. Thus, the categories $Sub_m(A)$, $Sub_{str}(A)$ and $Sub_{reg}(A)$ are closed under products in $\mathcal{A} \downarrow A$.

Hence, by the Slice and the Subcategory Theorem, they are locally finitely presentable, provided that they are closed under directed colimits, too.

Since the forgetful functor $\mathcal{A} \downarrow A \to \mathcal{A}$ creates directed colimits, this will be the case if *Mono*, $Mono_{str}$, and $Mono_{reg}$ respectively are closed under directed colimits.

This clearly holds for $Mono_{reg}$ (in any category) and for *Mono* in any l.f.p. category, since here directed colimits commute with limits.

It has, however, been shown by Adámek, Hébert and Sousa (2009), that this does *not* hold in general for strong monomorphims.

Since the forgetful functor $\mathcal{A} \downarrow A \to \mathcal{A}$ creates directed colimits, this will be the case if *Mono*, $Mono_{str}$, and $Mono_{reg}$ respectively are closed under directed colimits.

This clearly holds for $Mono_{reg}$ (in any category) and for *Mono* in any l.f.p. category, since here directed colimits commute with limits.

It has, however, been shown by Adámek, Hébert and Sousa (2009), that this does *not* hold in general for strong monomorphims.

Since the forgetful functor $\mathcal{A} \downarrow A \to \mathcal{A}$ creates directed colimits, this will be the case if *Mono*, $Mono_{str}$, and $Mono_{reg}$ respectively are closed under directed colimits.

This clearly holds for $Mono_{reg}$ (in any category) and for *Mono* in any l.f.p. category, since here directed colimits commute with limits.

It has, however, been shown by Adámek, Hébert and Sousa (2009), that this does *not* hold in general for strong monomorphims.

Answers

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

We, thus, have got

Theorem

For any object A in locally finitely presentable category $\mathcal{A}$ both, the subobjects of A, and the regular subobjects of A, form an algebraic lattice.

What kind of quotients?

While in a variety *quotient* clearly means "quotient presented by a strong (= regular) epimorphism" this is not so in arbitrary l.f.p. categories. Here are at least the following natural options

1. quotient presented by an *epimorphism*,
2. quotient presented by a *strong (extremal) epimorphism*,
3. quotient presented by a *regular epimorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi, strong mono) factorization structure; e.g. in **Poset** the natural quotients are those presented by regular (= strong) epimorphisms, while the image factorization of a monotone map is the (epi, strong mono) factorization.

Also, strong and regular epimorphisms need not coincide (see e.g. **Cat**).

What kind of quotients?

While in a variety *quotient* clearly means “quotient presented by a strong (= regular) epimorphism” this is not so in arbitrary l.f.p. categories. Here are at least the following natural options

1. quotient presented by an *epimorphism*,
2. quotient presented by a *strong (extremal) epimorphism*,
3. quotient presented by a *regular epimorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi, strong mono) factorization structure; e.g. in **Poset** the natural quotients are those presented by regular (= strong) epimorphisms, while the image factorization of a monotone map is the (epi, strong mono) factorization.

Also, strong and regular epimorphisms need not coincide (see e.g. **Cat**).

What kind of quotients?

While in a variety *quotient* clearly means "quotient presented by a strong (= regular) epimorphism" this is not so in arbitrary l.f.p. categories. Here are at least the following natural options

1. quotient presented by an *epimorphism*,
2. quotient presented by a *strong (extremal) epimorphism*,
3. quotient presented by a *regular epimorphism*.

Recall that a l.f.p. category admits a (strong epi, mono) and an (epi, strong mono) factorization structure; e.g. in **Poset** the natural quotients are those presented by regular (= strong) epimorphisms, while the image factorization of a monotone map is the (epi, strong mono) factorization.
Also, strong and regular epimorphisms need not coincide (see e.g. **Cat**).

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Notation

Motivation
Subobject lattices
Quotient lattices
The lattice of internal equivalence relations
Examples

For an $\mathcal{A}$-object A we denote by

1. $Quot_e(A)$ the full subcategory of $A \downarrow \mathcal{A}$ spanned by all epimorphims $A \xrightarrow{e} B$,
2. $Quot_{str}(A)$ the full subcategory of $A \downarrow \mathcal{A}$ spanned by all strong epimorphims $A \xrightarrow{e} B$,
3. $Quot_{reg}(A)$ the full subcategory of $A \downarrow \mathcal{A}$ spanned by all regular epimorphims $A \xrightarrow{e} B$.

Again, the poset of *all* (all *strong*, all *regular*) quotients of A is a skeleton of $Quot_e(A)$, $Quot_{str}(A)$ and $Quot_{reg}(A)$, respectively.

Questions

Again, we are to answer the following questions for an object A in a l.f.p. category $\mathcal{A}$

1. Is $Quot_e(A)$ locally finitely presentable?
2. Is $Quot_{str}(A)$ locally finitely presentable?
3. Is $Quot_{reg}(A)$ locally finitely presentable?

Note that we cannot proceed in analogy to the case of subobjects, since none of these categories is closed in $A \downarrow \mathcal{A}$ under products in general.

Since the forgetful functor $A \downarrow \mathcal{A} \to \mathcal{A}$ creates limits, this would mean that the morphism $K \xrightarrow{e} \prod_i A_i$ induced by a family of (strong, regular) epimorphisms $K \xrightarrow{e_i} A_i$ should be a (strong, regular) epimorphism again (which not even holds in **Set**).

Since, dually to the case of subobjects, the categories $Quot_e(A)$, $Quot_{str}(A)$ and $Quot_{reg}(A)$ are cocomplete, it remains to find, in each of them, a set $\mathcal{P}$ of finitely presentable objects, such that every object is a colimit of a directed diagram in $\mathcal{P}$.

This requires the following lemmata.

Note that we cannot proceed in analogy to the case of subobjects, since none of these categories is closed in $A \downarrow \mathcal{A}$ under products in general.

Since the forgetful functor $A \downarrow \mathcal{A} \to \mathcal{A}$ creates limits, this would mean that the morphism $K \xrightarrow{e} \prod_i A_i$ induced by a family of (strong, regular) epimorphisms $K \xrightarrow{e_i} A_i$ should be a (strong, regular) epimorphism again (which not even holds in **Set**).

Since, dually to the case of subobjects, the categories $Quot_e(A)$, $Quot_{str}(A)$ and $Quot_{reg}(A)$ are cocomplete, it remains to find, in each of them, a set $\mathcal{P}$ of finitely presentable objects, such that every object is a colimit of a directed diagram in $\mathcal{P}$.

This requires the following lemmata.

Lemma

The forgetful functor $A \downarrow \mathcal{A} \to \mathcal{A}$ creates connected (hence directed) colimits.

Lemma

The categories $Quot_e(A)$, $Quot_{str}(A)$, and $Quot_{reg}(A)$ are closed in $A \downarrow \mathcal{A}$ under directed colimits.

For $Quot_{reg}(A)$ this is obvious; for the other cases one uses the (proof of the) preceding lemma, where the case of strong epis requires that strong and extremal epis coincide.

Lemma

The forgetful functor $A \downarrow \mathcal{A} \to \mathcal{A}$ creates connected (hence directed) colimits.

Lemma

The categories $Quot_e(A)$, $Quot_{str}(A)$, and $Quot_{reg}(A)$ are closed in $A \downarrow \mathcal{A}$ under directed colimits.

For $Quot_{reg}(A)$ this is obvious; for the other cases one uses the (proof of the) preceding lemma, where the case of strong epis requires that strong and extremal epis coincide.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Lemma

The forgetful functor $A \downarrow \mathcal{A} \to \mathcal{A}$ creates connected (hence directed) colimits.

Lemma

The categories $Quot_e(A)$, $Quot_{str}(A)$, and $Quot_{reg}(A)$ are closed in $A \downarrow \mathcal{A}$ under directed colimits.

For $Quot_{reg}(A)$ this is obvious; for the other cases one uses the (proof of the) preceding lemma, where the case of strong epis requires that strong and extremal epis coincide.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Thus, we need to find sets $\mathcal{P}$ of finitely presentables in the respective categories of quotients, such that each such quotient is a directed colimit in $\mathcal{A}$ of morphisms from $\mathcal{P}$.

For strong epimorphisms this has been done by Adámek, Hébert and Sousa (2009). Thus

Proposition

For every object A in a l.f.p. category $\mathcal{A}$ the category $Quot_{str}$ is locally finitely presentable.

Hébert (2007) gave an example showing that this — not unexpectedly — cannot be achieved in general for epimorphims.

Thus, we need to find sets $\mathcal{P}$ of finitely presentables in the respective categories of quotients, such that each such quotient is a directed colimit in $\mathcal{A}$ of morphisms from $\mathcal{P}$.

For strong epimorphisms this has been done by Adámek, Hébert and Sousa (2009). Thus

Proposition

For every object A in a l.f.p. category $\mathcal{A}$ the category $Quot_{str}$ is locally finitely presentable.

Hébert (2007) gave an example showing that this — not unexpectedly — cannot be achieved in general for epimorphims.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Thus, we need to find sets $\mathcal{P}$ of finitely presentables in the respective categories of quotients, such that each such quotient is a directed colimit in $\mathcal{A}$ of morphisms from $\mathcal{P}$.

For strong epimorphisms this has been done by Adámek, Hébert and Sousa (2009). Thus

Proposition

For every object A in a l.f.p. category $\mathcal{A}$ the category $Quot_{str}$ is locally finitely presentable.

Hébert (2007) gave an example showing that this — not unexpectedly — cannot be achieved in general for epimorphims.

Regular quotients

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

The following requires some essentially straight forward caluclations.

Proposition

1. Let $f, g\colon P \to A$ be morphisms in an arbitrary category $\mathcal{A}$ and $A \xrightarrow{q} Q$ its coequalizer.
   Then $A \xrightarrow{q} Q$ is finitely presentable in $A \downarrow \mathcal{A}$, provided that P is finitely presentable in $\mathcal{A}$.
2. If $\mathcal{A}$ is l.f.p., then every regular epimorphism $A \xrightarrow{e} B$ is a directed colimit (in $\mathcal{A}$) of such morphisms.

Regular quotients

The following requires some essentially straight forward caluclations.

Proposition

1. Let $f, g\colon P \to A$ be morphisms in an arbitrary category $\mathcal{A}$ and $A \xrightarrow{q} Q$ its coequalizer.
   Then $A \xrightarrow{q} Q$ is finitely presentable in $A \downarrow \mathcal{A}$, provided that P is finitely presentable in $\mathcal{A}$.
2. If $\mathcal{A}$ is l.f.p., then every regular epimorphism $A \xrightarrow{e} B$ is a directed colimit (in $\mathcal{A}$) of such morphisms.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Since $Quot_{reg}$ is closed under directed colimits in $A \downarrow \mathcal{A}$, the finitely presentables in $A \downarrow \mathcal{A}$ described above are finitely presentable in $Quot_{reg}$, and they form a set, if $\mathcal{A}$ is l.f.p.. Thus

Corollary

For every object A in a l.f.p. category $\mathcal{A}$ the category $Quot_{reg}$ is locally finitely presentable.

Answers

Summarized we have got

Theorem

For any object A in locally finitely presentable category $\mathcal{A}$ both, the extremal quotients of A, and the regular quotients of A, form an algebraic lattice.

Kernel pairs vs equivalence relations

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

The result of the previous section on regular quotients can equivalently be expressed as

The kernel pairs on any object in a l.f.p. category $\mathcal{A}$ form an algebraic lattice.

While this, in case $\mathcal{A}$ is even a variety, is equivalent to the familiar result

The equivalence relations on any algebra in a variety $\mathcal{A}$ form an algebraic lattice.

in an arbitrary l.f.p. category $\mathcal{A}$ it is not!

Kernel pairs vs equivalence relations

The result of the previous section on regular quotients can equivalently be expressed as

The kernel pairs on any object in a l.f.p. category $\mathcal{A}$ form an algebraic lattice.

While this, in case $\mathcal{A}$ is even a variety, is equivalent to the familiar result

The equivalence relations on any algebra in a variety $\mathcal{A}$ form an algebraic lattice.

in an arbitrary l.f.p. category $\mathcal{A}$ it is not!

A further result

Theorem

The equivalence relations on any object A in a l.f.p. category $\mathcal{A}$ form an algebraic lattice.

Again, one only needs to show that the category of equivalence relations on A is closed in $Sub_m(A \times A)$ under intersections in $\mathcal{A}$, and under directed colimits. And here only transitivity requires some more serious calculations.

Examples

1. The lattice of subobjects of an algebra in a finitary variety forms an algebraic lattice.
2. The lattice of congruences of an algebra in a finitary variety form an algebraic lattice.
3. For every finitary **Set**-functor T the subalgebras of any T-algebra form an algebraic lattice.
4. For every polynomial (not necessary finitary) **Set**-functor T the sub-coalgebras of any T-coalgebra form an algebraic lattice.
5. For every polynomial (not necessary finitary) **Set**-functor T the quotients, and the regular congruences respectively, of any T-coalgebra form an algebraic lattice.

Examples

1. The lattice of subobjects of an algebra in a finitary variety forms an algebraic lattice.
2. The lattice of congruences of an algebra in a finitary variety form an algebraic lattice.
3. For every finitary **Set**-functor T the subalgebras of any T-algebra form an algebraic lattice.
4. For every polynomial (not necessary finitary) **Set**-functor T the sub-coalgebras of any T-coalgebra form an algebraic lattice.
5. For every polynomial (not necessary finitary) **Set**-functor T the quotients, and the regular congruences respectively, of any T-coalgebra form an algebraic lattice.

Examples

1. The lattice of subobjects of an algebra in a finitary variety forms an algebraic lattice.
2. The lattice of congruences of an algebra in a finitary variety form an algebraic lattice.
3. For every finitary **Set**-functor T the subalgebras of any T-algebra form an algebraic lattice.
4. For every polynomial (not necessary finitary) **Set**-functor T the sub-coalgebras of any T-coalgebra form an algebraic lattice.
5. For every polynomial (not necessary finitary) **Set**-functor T the quotients, and the regular congruences respectively, of any T-coalgebra form an algebraic lattice.

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

Examples

1. The lattice of subobjects of an algebra in a finitary variety forms an algebraic lattice.
2. The lattice of congruences of an algebra in a finitary variety form an algebraic lattice.
3. For every finitary **Set**-functor T the subalgebras of any T-algebra form an algebraic lattice.
4. For every polynomial (not necessary finitary) **Set**-functor T the sub-coalgebras of any T-coalgebra form an algebraic lattice.
5. For every polynomial (not necessary finitary) **Set**-functor T the quotients, and the regular congruences respectively, of any T-coalgebra form an algebraic lattice.

A problem

In Examples 1., 3. and 4. the algebraic lattices in question obviously are even algebraic sublattices of the powerset lattice of the underlying set of the (co)algebra under consideration.

For coalgebras this can be generalized to (not necessarily finitary) **Set**-functors T which preserve intersections and, in fact characterizes these functors.

This rises the open question (J. Adámek) whether finitary **Set**-functors T can be characterized the same way, too, using subobject lattices of T-algebras.

A partial result: The (non finitary) powerset functor has algebras, whose subalgebras do not form an algebraic sublattice of the powerset lattice of the respective underlying set!

Motivation

Subobject lattices

Quotient lattices

The lattice of internal equivalence relations

Examples

A problem

In Examples 1., 3. and 4. the algebraic lattices in question obviously are even algebraic sublattices of the powerset lattice of the underlying set of the (co)algebra under consideration.

For coalgebras this can be generalized to (not necessarily finitary) **Set**-functors T which preserve intersections and, in fact characterizes these functors.

This rises the open question (J. Adámek) whether finitary **Set**-functors T can be characterized the same way, too, using subobject lattices of T-algebras.

A partial result: The (non finitary) powerset functor has algebras, whose subalgebras do not form an algebraic sublattice of the powerset lattice of the respective underlying set!

A problem

In Examples 1., 3. and 4. the algebraic lattices in question obviously are even algebraic sublattices of the powerset lattice of the underlying set of the (co)algebra under consideration.

For coalgebras this can be generalized to (not necessarily finitary) **Set**-functors T which preserve intersections and, in fact characterizes these functors.

This rises the open question (J. Adámek) whether finitary **Set**-functors T can be characterized the same way, too, using subobject lattices of T-algebras.

A partial result: The (non finitary) powerset functor has algebras, whose subalgebras do not form an algebraic sublattice of the powerset lattice of the respective underlying set!

A problem

In Examples 1., 3. and 4. the algebraic lattices in question obviously are even algebraic sublattices of the powerset lattice of the underlying set of the (co)algebra under consideration.

For coalgebras this can be generalized to (not necessarily finitary) **Set**-functors T which preserve intersections and, in fact characterizes these functors.

This rises the open question (J. Adámek) whether finitary **Set**-functors T can be characterized the same way, too, using subobject lattices of T-algebras.

A partial result: The (non finitary) powerset functor has algebras, whose subalgebras do not form an algebraic sublattice of the powerset lattice of the respective underlying set!

Thanks for your attention!