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Difference hierarchies over lattices

The notion of a difference hierarchy, first introduced by Hausdorff, plays an important role in many areas of mathematics, logic and theoretical computer science such as in descriptive set theory, complexity theory, and in the theory of regular languages and automata. From a lattice theoretic point of view, the difference hierarchy over a bounded distributive lattice stratifies the Boolean algebra generated by it according to the minimum length of difference chains required to describe each Boolean element. While each Boolean element has a description by a finite difference chain, there is no canonical such writing in general. In this talk I will start by explaining why, in the case where the lattice admits a (co-)Heyting structure, each Boolean element over the lattice has a canonical minimum length decomposition into a Hausdorff difference. Using a compactness argument via canonical extensions, this may be extended to every lattice that is the direct limit of the images of a family of maps $\{g_i : S_i \to B\}_{i\in I}$ admitting an upper adjoint, where $\{S_i\}_{i\in I}$ is a family of semilattices and B a Boolean algebra. In particular, this provides a topological proof of the well-known fact that every element b in the Booleanization of a bounded distributive lattice D may be written as a difference

$$b = a_1 - (a_2 - (\ldots(a_{n-1} - a_n)\ldots)),$$

where $a_1 \geq a_2 \geq \ldots \geq a_{n-1} \geq a_n$ are elements of D.

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