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Order theory, enriched

Since Lawvere's [8] rather surprising presentation of metric spaces as enriched categories – or, more modestly, as enriched ordered sets, several attempts were made to investigate metric spaces as "enriched ordered sets". As examples, we mention here the works on "metric domain theory" presented in [4, 3, 12] and – using approach spaces (a metric version of topological spaces) – in [13, 5, 9], as well as the papers [2, 7, 1] generalising classical "order-theoretic" duality results to the metric context. In this talk we report on our ongoing work in this direction, in particular on "Halmos-Stone-type" duality results initiated in [6]. We discuss metric variants of lattices and Boolean algebras together with their duality theory, which naturally forces us to study metric compact Hausdorff spaces (a metric version of Nachbin's ordered compact Hausdorff spaces [10]), a metric version of the classic Vietoris space (in fact, monad), and also to employ the theory of approach frames (a metric version of frames) developed in [11, 2].

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