Tomáš Jakl

Laboratoire J.A. Dieudonné, Université Côte d'Azur, France

A pointfree account of Carathéodory's Extension Theorem

Carathéodory's extension theorem says that any σ-finite finitely additive measure $\mu\colon \Sigma \to \mathbb{R}$ on a Boolean algebra of subsets $\Sigma \subseteq \mathcal{P}(X)$ can be uniquely extended to a measure on the smallest σ-algebra $\sigma(\Sigma)$ containing Σ. In the proof, μ is first extended to an outer measure $\mu^*\colon \mathcal{P}(X) \to \mathbb{R}$ and then the resulting measure is obtained as the restriction of μ^* to $\sigma(\Sigma)$.

In the point-free context, one starts with a σ-finite (abstract) finitely additive measure $\mu\colon B \to \mathbb{R}$, for some Boolean algebra B. Then, in order to extend μ to an outer measure, one needs to find an ambient Boolean frame which contains B and has the same set of points as the spectrum of B. It turns out that the required Boolean frame can be obtained by the frame-theoretic discretisation construction $\mathcal{S}_\mathfrak{c}(-)$, recently studied by Pultr, Ball, Picado, Tozzi, etc. As a result, we can prove Carathéodory's theorem in a purely point-free fashion.

Furthermore, we also discuss the relationship between $\mathcal{S}_\mathfrak{c}(-)$ and canonical extensions, which play the role of discretisation in the context of lattices.

References

[1] Ball R., Pultr A., *Maximal essential extensions in the context of frames*, Algebra Universalis (2018) 79:2–32.

[2] Gehrke M., Jónsson B., *Bounded distributive lattices with operators*, Mathematica Japonica (1994) 40:207–215 .

[3] Jónsson B., Tarski A., *Boolean algebras with operators I.*, American Journal of Mathematics (1951) 73:891–939.

[4] Picado J., Pultr A., Tozzi A., *Joins of closed sublocales*, Houston Journal of Mathematics (to appear).

The research discussed has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No.670624).