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Bitopological sublocales

In the category **Top** of topological spaces, subspaces of a space X are extremal monomorphisms $f : Y \hookrightarrow X$, since these are the continuous injections such that the structure of Y is refined enough to be a subspace of X. To obtain a pointfree analogue of this notion we dualize this and obtain that sublocales of a certain locale L are extremal epimorphisms in **Frm** from L. These are equivalently characterized as congruences on L or frame surjections from L. Given a relation R on a frame L it is quite straightforward to describe the smallest congruence induced by it.

There are several situations when given a topological space X it is quite elegant and convenient to consider the opens $\Omega(X)$ as being the union of two separate topologies on X. This might happen for instance when the specialization order on X is nontrivial, the two topologies being that of open upsets and downsets. This is the motivation behind *bitopological spaces*, sets equipped with two topologies. These, equipped with functions continuous in both topologies, form the category **biTop**. As order-theoretical, pointfree duals of bitopological spaces we have d-frames, quadruples $(L^+, L^-, \mathrm{con}, \mathrm{tot})$, where L^+ and L^- are frames, and con and tot are two relations on the product $L^+ \times L^-$ satisfying certain axioms. These form a category **dFrm** when equipped with suitable morphisms. We will introduce this category and motivate why this is a suitable dual of **biTop**.

In the category **dFrm** the duals of subspace embeddings in **biTop** still are extremal epimorphisms. These are frame surjections from $L^+ \times L^-$ satisfying an extra condition involving con and tot. This condition translates as a restriction on the congruences on $L^+ \times L^-$ that give raise to extremal epis in **dFrm**. We call these suitable congruences *reasonable*. Given a relation R on $L^+ \times L^-$ the extremal epi induced by it is the minimal reasonable congruence containing R. This is not easy to describe, and in general we need an iterative process of transfinitely many steps.

In my talk I will describe several examples where we actually have a concise and explicit description of the reasonable congruence induced by some relation R on $L^+ \times L^-$. In particular this will allow a concrete description of bitopological analogues of open and closed sublocales, and of Booleanization. Classes of d-frames for which we can provide an explicit desciption of all three these kinds of subocales include: d-frames where the relations con and tot are minimal, d-frames $(L^+, L^-, \mathrm{con}, \mathrm{tot})$ where both L^+ and L^- are linear, the so called *complemented* d-frames.

References

[1] T. Jakl, d-Frames as algebraic duals of bitopological spaces, Ph.D. Thesis.

[2] T. Jakl, A. Jung, A. Pultr, Quotients of d-frames, Submitted.

Joint work with Achim Jung.