The manifestation of Hilbert's Nullstellensatz in Lawvere's Axiomatic Cohesion

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Let $(\mathcal{D}, \mathtt{a})$ be an atomic site and $\mathbf{Shv}\,(\mathcal{D}, \mathtt{a}) \to \widehat{\mathcal{D}}$ be the associated sheaf topos. Any functor $\phi : \mathcal{C} \to \mathcal{D}$ induces a geometric morphism $\widehat{\mathcal{C}} \to \widehat{\mathcal{D}}$ and, by pulling-back along $\mathbf{Shv}\,(\mathcal{D}, \mathtt{a}) \to \widehat{\mathcal{D}}$, a geometric morphism $q : \mathcal{F} \to \mathbf{Shv}\,(\mathcal{D}, \mathtt{a})$. We give a sufficient condition on ϕ for q to satisfy the Nullstellensatz (and Sufficient Cohesion) in the sense of [1]. This is motivated by the examples where $\mathcal{D}^{op}$ is a category of finite field extensions. In this case, the sufficient condition holds thanks to Hilbert's Nullstellensatz.

References

[1] F. W. Lawvere, *Axiomatic Cohesion*, TAC (2007).