The manifestation of Hilbert’s Nullstellensatz
in Lawvere’s Axiomatic Cohesion

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Let \((\mathcal{D}, a)\) be an atomic site and \(\text{Shv}(\mathcal{D}, a) \to \hat{\mathcal{D}}\) be the associated sheaf topos. Any functor \(\phi : \mathcal{C} \to \mathcal{D}\) induces a geometric morphism \(\hat{\mathcal{C}} \to \hat{\mathcal{D}}\) and, by pulling-back along \(\text{Shv}(\mathcal{D}, a) \to \hat{\mathcal{D}}\), a geometric morphism \(q : \mathcal{F} \to \text{Shv}(\mathcal{D}, a)\). We give a sufficient condition on \(\phi\) for \(q\) to satisfy the Nullstellensatz (and Sufficient Cohesion) in the sense of [1]. This is motivated by the examples where \(\mathcal{D}^{\text{op}}\) is a category of finite field extensions. In this case, the sufficient condition holds thanks to Hilbert’s Nullstellensatz.

References