Reflections of universal algebras into idempotent subvarieties and their Galois theories

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A sufficient condition for the preservation of finite products by a reflector from a variety of universal algebras into an idempotent subvariety will be presented. This condition will be then stated in a more general setting. In reflections for which such condition holds, semi-left-exactness and the stronger stable units property are equivalent. It will be shown that simple and semi-left-exact reflections coincide in the context of universal algebras, and will be given characterizations of the classes in the derived reflective factorization systems. Several statements help to characterize covering and stably-vertical morphisms in universal algebras, and in the more general setting referred to above. The classes of separable, purely inseparable and normal morphisms will be characterized as well. The study of Galois descent morphisms and their Galois groupoids provides conditions under which these groupoids reduce to equivalence relations.

References

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