Efficient Paths

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1. Introduction

Road guidance is a relevant topic nowadays and it has produced many developments both in theory and practice. The initial approach with the single-objective shortest path problem, as used in navigation systems, is insufficient since we actually seek paths that minimize conflicting objectives, such as distance (or time) and cost (toll roads). The multi-objective shortest path problem formulation is used instead, focusing on the efficient paths, for which there is no path improving one objective without worsening another one. Unfortunately, this problem is NP-complete and intractable, that is, the number of efficient paths can grow exponentially with the number of the nodes in the network, even for the bi-objective case. However, both theoretical and experimental work suggest that the number of efficient paths is not that large and may be feasible in practice, which opens a new opportunity to enlarge the range of applications for navigation systems. In this project we aim to implement and analyse:

1) several label setting and label correcting algorithms as well as further speed-up extensions proposed in this project for the bi-objective shortest path problem
2) techniques from multicriteria decision making that help the user to choose the preferable solution from a set or a representative subset of efficient paths.

2. Problem definition

The multi-objective shortest path problem (MOSPP) is defined on a network $G=(N,A)$ where k costs are associated to each arc. In this network, two nodes are fixed, the source (s) and the target (t). We want to find paths from s to t that minimize, in each objective, the sum of the costs of its constituent arcs. Since generally there is conflict among the objectives, the utopian solution does not exist and we have to look for efficient paths. For these solutions, there is no path from s to t that improves the cost in one objective without worsening another one [Vin74]. More formally, given two paths p and q from s to t, and denoting by $f_i(p)$ the i-th component of the objective function (that is, the sum of the i-th cost over the arcs belonging to p) we have that:

1) *p dominates q* if and only if $f_i(p) \leq f_i(q)$, for all i in $\{1, ..., k\}$, with at least one strict inequality.
2) p is an efficient path if and only if there is no path q that dominates p.

Hence, solving the MOSPP means that the entire set of efficient paths has to be determined. Although it has been shown that the number of solutions can grow in an exponentially manner [Han80], in real or artificially generated instances this number generally does not increase so quickly [MW06].

After having solved the MOSPP problem, the question of how to present the set of efficient paths to the decision-maker remains. In fact, the presentation of a set with hundreds or thousands of efficient paths will not be useful to the decision-maker. The problem of choosing the most preferred solution from a set is well-studied in the field of multicriteria decision analysis. A possible manner is to present a subset of efficient paths to the user that contains some desirable properties in terms of spreadness, uniformity, coverage of the solutions in the objective space with a given cardinality [RW05].

3. Related work

In network related problems, we frequently have to determine the shortest path between two points (the shortest path problem). The study of this subject goes back to the 1950's [For56], being one of the first and also one of the most studied combinatorial optimization topics covered by Operations Research. In fact, the wide range of applications either directly or as a sub-problem of other combinatorial optimization problems [AMO93] have caught the attention of many individual researchers and companies interested in possible commercial applications. The growing up of our society leads to an increase in the amount of data to be dealt, brought with them new challenges and continues to keep it as an active topic nowadays [WW05, DGJ06, GKW07].

However, the shortest path does not actually fit entirely our needs because in many practice situations we intend to minimize several objectives simultaneously. It leads us to the multi-objective version (MOSPP) [Vin74], which is NP-complete [Ser86] and intractable [Han80]. The difficulty to deal with the MOSPP is mainly related with the number of efficient paths: it may be high, requiring a large computational effort to solve the problem. Nevertheless, some papers have shown that, in practice, the number of efficient paths is often small for the bi-objective version of this problem: The PI performed an empirical study about the evolution of the number of efficient paths on several kinds of artificial networks using polynomial regression models to fit the increasing of the number of efficient paths with the number of nodes, density, number of objectives and the cost range on cyclic networks [San03]; Paixão and Santos extend the previous results for acyclic and undirected networks [PRS05], which are available as a public library of benchmark instances for the MOSPP [wwwMOSPP] and mentioned in Beasley's OR Library [Bea90]; furthermore, Mueller-Hannemann and Weihe [MW06] state tractable bounds for the number of efficient paths on certain types of bi-objective networks.

The algorithms initially proposed to determine the full set of efficient paths [Vin74, Han80, Mar84, PS11] are based on generalizations of the labelling algorithm for the single-objective. The great weakness of these methods is the need to compute all the efficient paths starting at the initial node (to any other node in the network), since the number of efficient paths from the initial node to the terminal one is unknown beforehand. The two-phase method, for the bi-objective problem, computes in the first phase the supported efficient paths and next swapped restricted regions in the objective space using an adaptation of the labelling/ranking algorithms [CCC99]. A new approach based on ranking algorithms defines a stop ranking condition, which guarantees that the entire set of efficient paths can be determined without computing the set of efficient paths starting at the initial node (to any other node in the network). Although the original version [CM82] was not competitive with conventional approaches [MMO91], recent advances on ranking procedures [MPS99] (which are actually gold standard in the literature) and improvements done in the original version allowed us to define a new algorithm that outperforms the classical approaches [PS08, MRPS07].

Assuming that we can compute efficiently the entire set of efficient paths, it is not useful to confront the decision-maker with the full range of solutions when the number of paths is too large. In order to overcome this drawback, two strategies have been considered to look for a specific efficient path. In the first one, a utility function is used to score the efficient path and the one with lowest value is selected. The most popular utility function is the weighted sum of the objectives [DD97] because it is a linear function and simplifies the optimization process (it is equivalent to a SPP with a single objective). However, when a convex utility function is used (as in the weighted sum), the selected solution belongs to the frontier of the convex hull of the set of efficient paths, which usually contain a few percentage of efficient paths [ES03]. Efficient algorithm has also been defined for nonconvex utility functions like the Euclidean [PRS03] and the weighted Tchebycheff norm [HF08]. At any case, the role of the decision-maker is limited since he only acts in the beginning of the search procedure by defining the utility function. A second approach applies interactive decision support methods where the decision-maker guides the search of the desired solution into the objective space [GG03].

4. Proposed research

In this project we plan to implement the several existing algorithms proposed in the literature in a common programming framework and propose several extensions that may improve the run-time such as:

1) Use of additional information provided by bounds to discard further labels;
2) Extend bidirectional Dijkstra algorithm for the bi-objective case.

The technique in 1) is an innovative mixture of branch-and-bound strategies within the labelling algorithms. The technique in 2) is well-known for the single-objective case, and was never applied in the multi-objective context. Finally, we aim to apply techniques from multicriteria decision making [FGE05] to help the user to choose the most preferred solution from a reasonably small-sized subset of set of efficient paths. In particular, we focus on compact representation of the set of efficient paths, by finding subsets that have interesting properties, such as spreadness and uniformity/coverage with respect to a given cardinality that is choosen *a priori*. The problem of finding a subset with given properties can be formalized as a facility/location problem.

The benchmark data for testing will consider the networks described in [wwwMOSPP]. This benchmark contains thousands of networks with different properties (edge degree, number of nodes) and structures (grid, random and complete networks). Whenever possible, we will also consider real-life instances, for instance, free USA road instances available on http://www.dis.uniroma1.it/~challenge9/data/tiger/. Our focus is on the bi-objective version, which is, by far, the most interesting and manageable case from an application point of view. The final package will be available publicly through the internet as open source.

5. Budget requested

We would like to apply for a funding of 12k Euros, which will be used to fund one element of the team, Pedro Correia, a PhD student at the University of Coimbra, for 12 months. Pedro has a very large experience in programming and he will be in charge of the implementations. Some part of this work will be used on his PhD thesis. The remaining members have a large experience on exact and approximate algorithms for multiobjective optimization. In particular, the PI has several publications on the main topic of this project since his PhD in 2003.

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