Discrete and continuum modeling of biological network formation

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Motivated by recent papers describing rules for natural network formation in discrete settings, we propose an elliptic-parabolic system of partial differential equations. The model describes the pressure field due to Darcy's type equation and the dynamics of the conductance network under pressure force effects with a diffusion rate representing randomness in the material structure. We show how the PDE system is derived from the discrete graph-based model as a mean-field-type limit. Then we present several analytical results - existence of global weak solutions and of local mild solutions and their long term behavior. Moreover, we study the structure and stability properties of steady states that play a central role to understand the pattern capacity of the system. We show that patterns (network structures) occur in the regime of small material randomness. Moreover, we present results of systematic numerical simulations of the system that provide further insights into the properties of the network-type solutions.