# Synchronization phenomenon in complex systems

**Rosario D. Laureano**[1], **Diana A. Mendes**[2], **Manuel A. M. Ferreira**[1]

[1] ISCTE-IUL University Institute of Lisbon, Department of Mathematics
[2] ISCTE-IUL, Department of Quantitative Methods for Management and Economics
maria.laureano@iscte.pt, diana.mendes@iscte.pt,
manuel.ferreira@iscte.pt

Whenever two ”events” seem to happen simultaneously during an extended time period, the synchrony is probably not a coincidence. The first observations related to synchronization were reported by Huygens in 1665 [1]; it was indicated by the equal periods of coupled clocks.
Until a few years ago, the study of synchrony was dispersed by very different knowledge areas, such as biology, physics or economics, through disconnected works. After a seminal paper of Fujisaka and Yamada [2], the study of synchrony has been gradually constituted as a concrete and transverse research topic. By applying the theory of dynamical systems, the complex theory or the computer science, and using the available technology, it has been possible to understand the synchrony of ”events” as coupled oscillators. In fact, groups of fireflies, pacemaker cells, lasers, pendulums or planetary orbits are all collections of (living or inanimate) oscillators. Two (or more) oscillators are said to be coupled if an appropriate communication process (physical or chemical) allows some influence between them. This influence can be bidirectional (mutual) or unidirectional. There are many different channels, both in nature and in artificial environments, which play the role of coupling between systems.
The emergent field studying the synchronization phenomena is still largely unexplored. However, it is immediately interesting for its high potential application and actually includes processes where the oscillators are replaced by chaotic systems - *chaos synchronization* - or where they are coupled in less symmetrical ways or in intricate networks.
Coupled systems are constructed from simple, low-dimensional and form new and more complex organizations, with the belief that dominant features of the underlying components will be retained. This building up approach can also be used to create a novel system which behavior is more flexible or richer than that of the components, but which analysis and control remains tractable.

Even in the so called *identical (complete) synchronization* - when the coupling is between identical chaotic systems but starting from different initial conditions - the chaos synchronization phenomenon is especially challenging. In fact, the possibility of chaotic systems oscillate in a coherent and synchronized way is not an obvious phenomenon since the *sensitive dependence on initial conditions* (that is, any infinitesimal perturbations of the initial conditions lead to the exponential divergence of nearby starting orbits) is one of the main features associated with the chaotic behavior. However, when ensembles of chaotic systems are coupled, the attractive effect of a suitable coupling can counterbalance the trend of the trajectories to separate due to chaotic dynamics.
The effectiveness of a coupling between systems with equal dimension follows of the analysis of the synchronization error. By mean of an appropriate coupling, it is possible to reach full, *asymptotical* or *practical synchronization*, depending on the coupling degree. In chaos synchronization we seek subspaces of the coupled system space - the *synchronization set* - in which a special kind of motion, which relates the coupled system, takes place.
Other synchronization regimes, in addition to the identical synchronization, are the *generalized synchronization* and the *phase synchronization*. When the systems in coupling are distinct - if they simply have different sets of parameters or even by having different dimension - the process is called generalized synchronization. It is a regime of synchronization where exists a one-to-one smooth mapping between oscillations of each system. Hence, knowing the state of one system enables us to know the state of the other system. Phase synchronization is defined as the appearance of a certain relationship between the phases of the systems in coupling while the amplitudes can remain uncorrelated.

Since the work of Pecora and Carroll [3] various coupling methods and several news concepts necessary for analyzing chaos synchronization have been developed.

In our work it was considered the synchronization phenomena of (identical or nonidentical) chaotic dynamical systems, with nonlinear unidirectional and bidirectional coupling schemes, both in continuous and discrete time [4]. In order to illustrate the coupling methods, we always use a system of two coupled chaotic systems.

In continuous time, for identical and generalized synchronization, we apply various unidirectional and bidirectional coupling schemes between Lorenz, Rössler or hyperchaotic Rössler systems with control parameters that lead to chaotic behavior. We combine several coupling schemes - diffusive linear bidirectional coupling, unidirectional coupling by control function, unidirectional coupling by dislocated negative feedback control or active-passive decomposition for several driver signals - with total or partial replacement on the nonlinear terms of the second system, a coupling version that was less explored. In some cases we only conclude about local stability of the synchronous state; the effectiveness of a coupling between chaotic systems with equal dimension follows of the analysis of the synchronization error. In other coupling schemes the sufficient conditions of global stable synchronization are guaranteed from a different approach of the Lyapunov direct method for the transversal system.

In discrete time, we study a nonlinear coupling scheme that appears in natural a family of analytic complex quadratic maps and an We are not aware about any studies of this type of coupling. It is an asymmetric coupling between two real quadratic maps. When practical synchronization is not achieved, but the difference between the dynamical variables of the systems is limited, we still can apply a chaos control technique. We obtain stable identical and generalized synchronization with some versions of the original coupling, highlighting the absence of symmetry. Two of them are generalizations promoting the use of different parameters coupling. By analyzing the difference between the dynamical variables of the systems, we obtain some results leading to stable synchronization.

**Keywords:** complex systems; chaos synchronization; global stability; Lyapunov direct method; Lorenz system, Rössler system; hyperchaotic Rössler system
**Acknowledgments.** Partially supported by the strategic project of ISCTE-IUL.

## References

[1] Ch. Huygens, *LaTeX: Horoloquium Oscillatorium*. Apud. F. Muguet, Paris, 1673.
[2] H. Fujisaka, T. Yamada, Stability theory of synchronized motion in coupled oscillator systems, *Progress of Theoretical Physics*, **69**(1), pp. 32-47, 1983.
[3] L.M.. Pecora, T.L. Carroll, Synchronization in chaotic systems, *Physical Review Letters*, **64**(8), pp. 821-824, 1990.
[4] R.. Laureano, D.A. Mendes, Dislocated Feedback Control with Partial Replacement between Chaotic Systems, *Mathematica Aeterna*, **3**(5), pp. 337-348, 2013.