Fibrewise injectivity and Kock-Zöberlein monads

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Abstract. Using Escardó-Flagg approach to injectivity via Kock-Zöberlein monads in $T0$ topological spaces [3], and Hofmann's recent study of injectivity for spaces [4], we characterize continuous maps which are injective with respect to special classes of embeddings using convergence: see [1]. In fact, convergence has been shown to be very useful in the characterization of special classes of maps, like effective descent, exponentiable and triquotient maps, but for injective continuous maps such a characterization was missing. Further, we illustrate how this approach may be a step towards a fibrewise version of Scott's characterization of injective topological spaces as continuous lattices [5].

Finally, we investigate fibrewise injectivity in more general settings, using results of [2].

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[4] D. Hofmann, A four for the price of one duality principle for distributive topological spaces, preprint, arXiv:math.GN/1102.2605.

[5] D. Scott, Continuous lattices, in: *Springer Lecture Notes Math.* 274 (1972), pp. 97-136.